

**CEE 536
CRITICAL PATH METHODS**

EXAMPLE PROBLEMS



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1. NETWORK CONSTRUCTION

- ACTIVITY ON ARROW
- ACTIVITY ON NODE

Problem 1.1

- a) Construct an activity on arrow network based on the activity descriptions below. Show all your work. Label activities in the network by their activity letters and node numbers. Remove any redundant dependencies and label dummy activities DUMMY1, DUMMY2, etc.
- b) Construct a precedence network based on the same activity descriptions below. Show all your work. Label activities in the network by their activity letters and node numbers. Remove all redundant dependencies and arrange activities in proper sequence steps.

- Activities H, R2, T1 start the project.
- Activity T2 can start when Activities H, E1 and S are completed.
- Activity E1 also depends on Activity R2.
- Activity X follows Activity H and precedes Activity L.
- Activity E is preceded by Activities T2 and P1.
- The predecessors to Activity G are Activities L, T2 and P1.
- The successors to Activity T1 are Activities E1, S, W and D2.
- Activity P1 cannot begin until Activity W is finished.
- Activity P2 and F follow Activities W and D2, and precede Activities E and R1.
- Activity O2 depends on T2 and P1 , and precedes Activity L.

Problem 1.2

- a) Construct an activity on arrow network based on the activity descriptions below. Show all your work. Label activities in the network by their activity letters and node numbers. Remove any redundant dependencies and label dummy activities DUMMY1, DUMMY2, etc.
- b) Construct a precedence network based on the same activity descriptions below. Show all your work. Label activities in the network by their activity letters and node numbers. Remove all redundant dependencies and arrange activities in proper sequence steps.

- Activity I follows Activity B and precedes Activity Q.
- Activity B1 precedes Activity P and follows the completion of Activities Q , K2 , and E.
- Activity R follows the completion of Activity B.
- Activity S2 follows Activities R and S1, and precedes Activity P.
- Activity K3 is preceded by Activities X , L , and Z, and followed by Activities G and F.
- Activity E2 precedes Activities A1 , X , L , and Z.
- Activity B can start when Activities A1 and X are completed.
- The predecessors to Activity S1 are Activities E , G , and F.
- Activity E depends on Activity L and E2 and precedes Activities N2, S1 , and K2.
- Activity K2 follows Activities N2 , R , and L.
- Activity P depends on Activities R and N2.
- Activity S2 depends on Activities X, F, and E2.

Problem 1.3

- a) Construct an activity on arrow network based on the activity descriptions below. Show all your work. Label activities in the network by their activity letters and node numbers. Remove any redundant dependencies and label dummy activities DUMMY1, DUMMY2, etc.
- b) Construct a precedence network based on the same activity descriptions below. Show all your work. Label activities in the network by their activity letters and node numbers. Remove all redundant dependencies and arrange activities in proper sequence steps.

The predecessors to Activity Z2 are Activities L , C and R.

The successors to Activity B are Activities E1 , S , W and D2.

Activity E1 also depends on Activity M.

Activity U and F follow Activities W and D2 , and precede Activities E and R1.

Activity Y follows Activities C and R , and followed by Activity L.

Activities D , M , and B start the project.

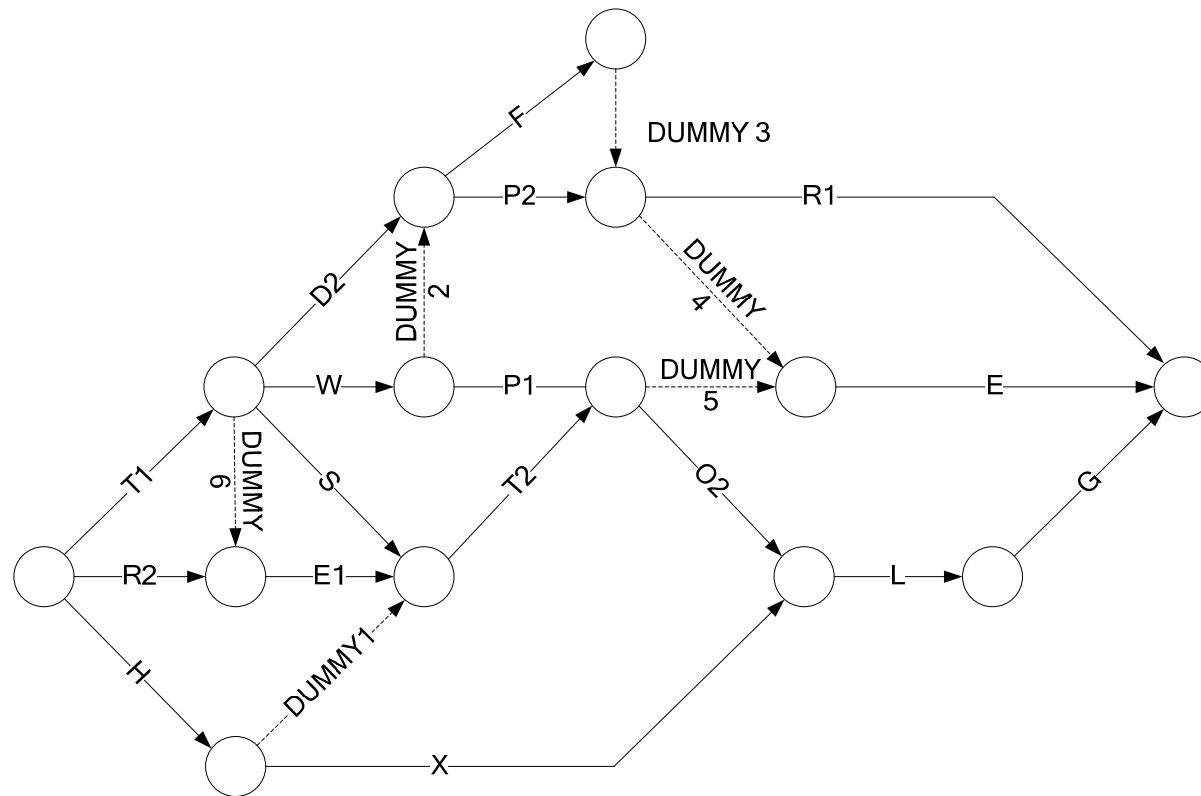
Activity C can start when Activities D , E1 and S are completed.

Activity R cannot begin until Activity W is finished.

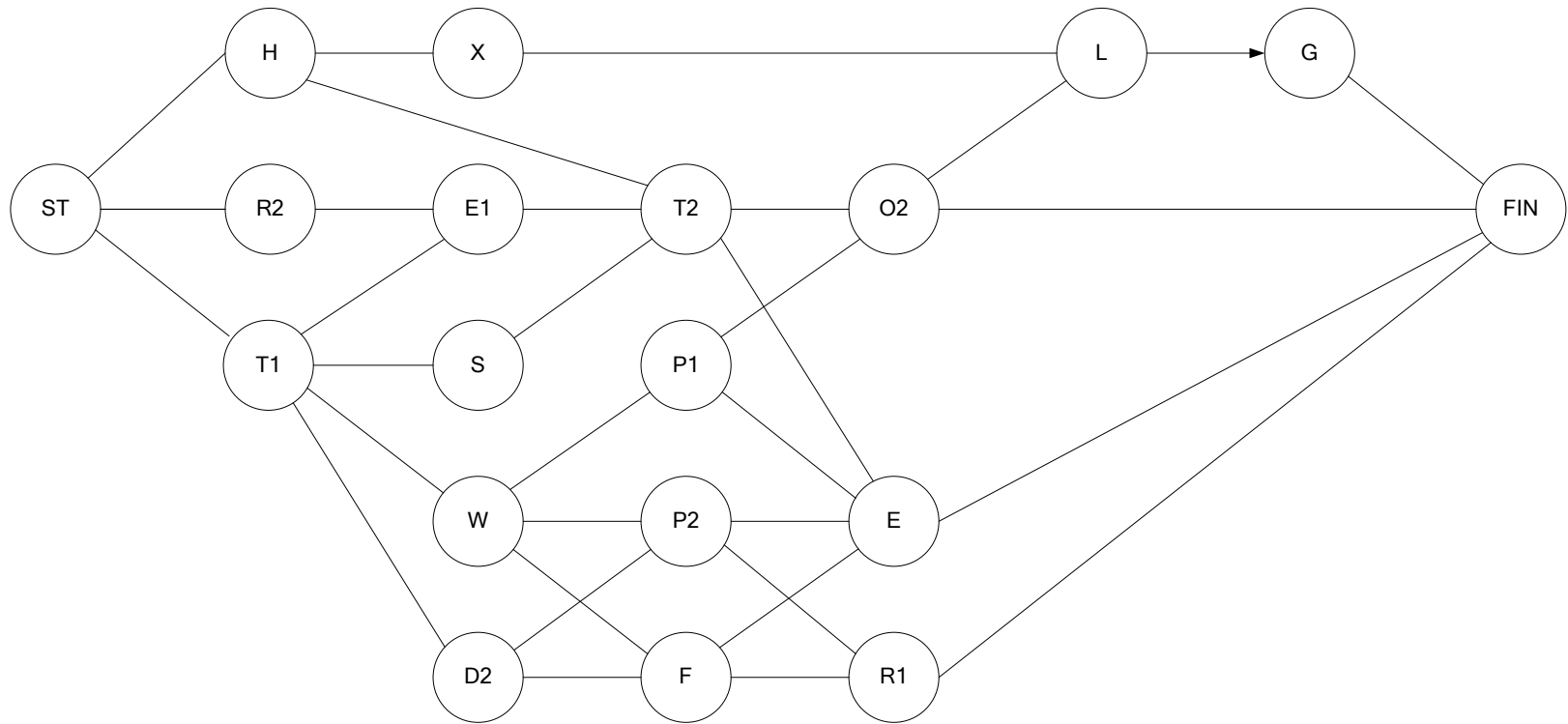
Activity I follows Activity D and precedes Activity L.

Activity E follows Activities C and R.

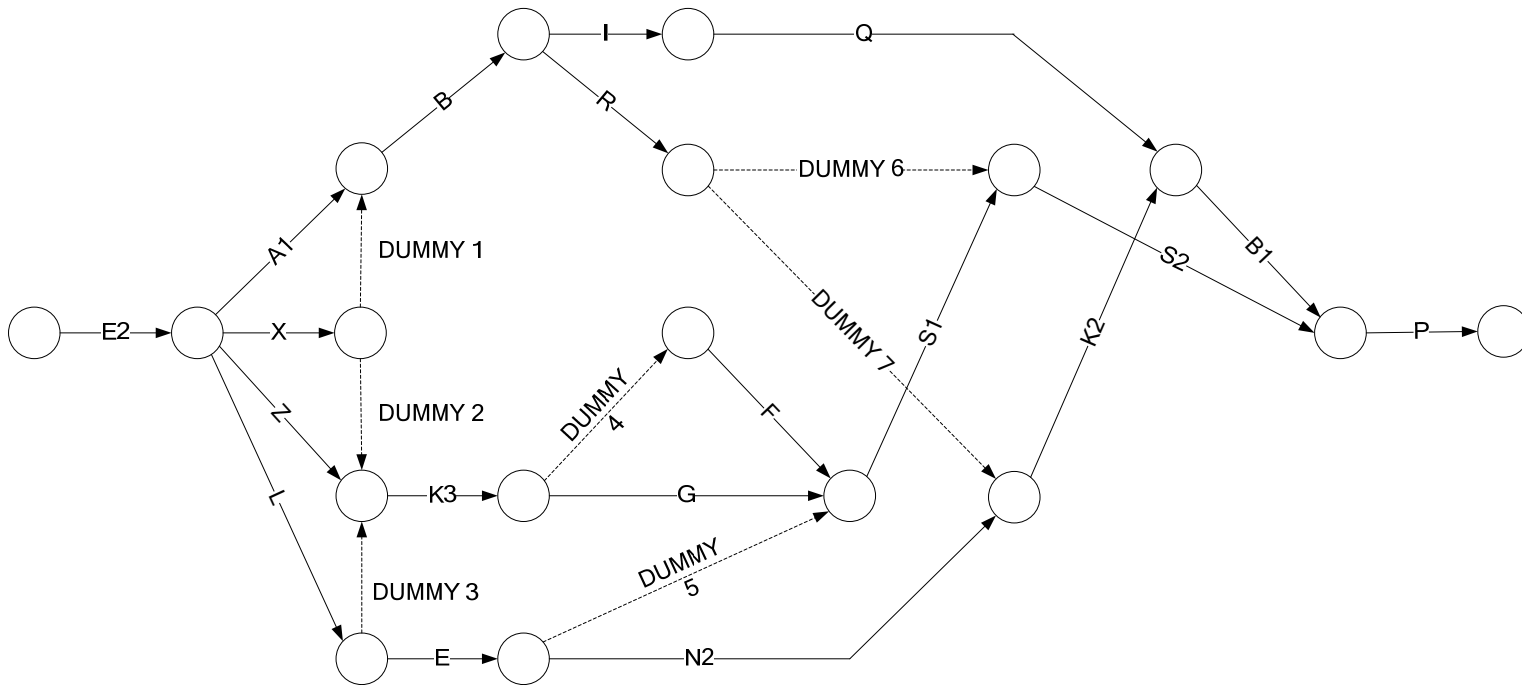
Solution 1.1.a



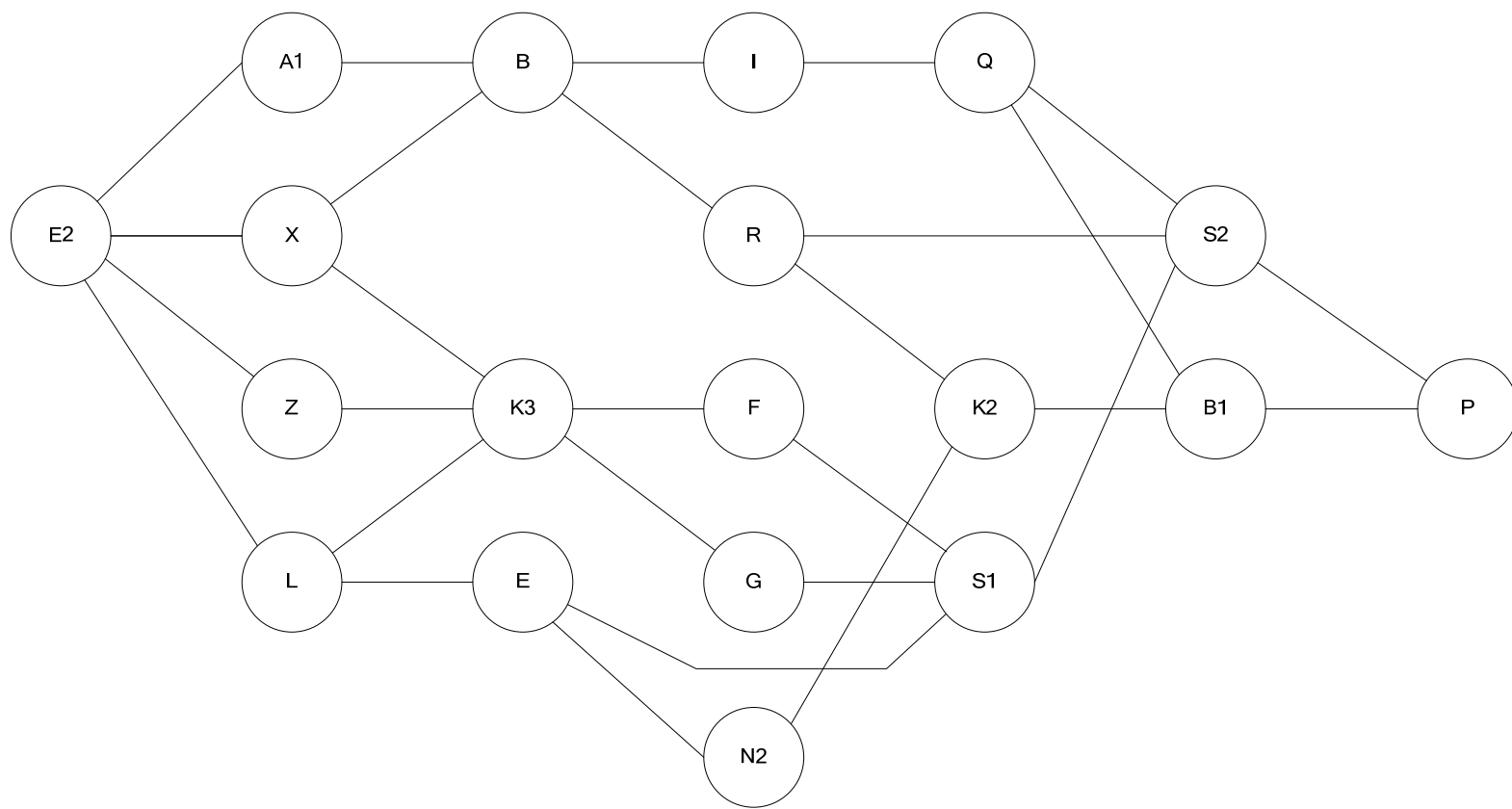
Solution 1.1.b



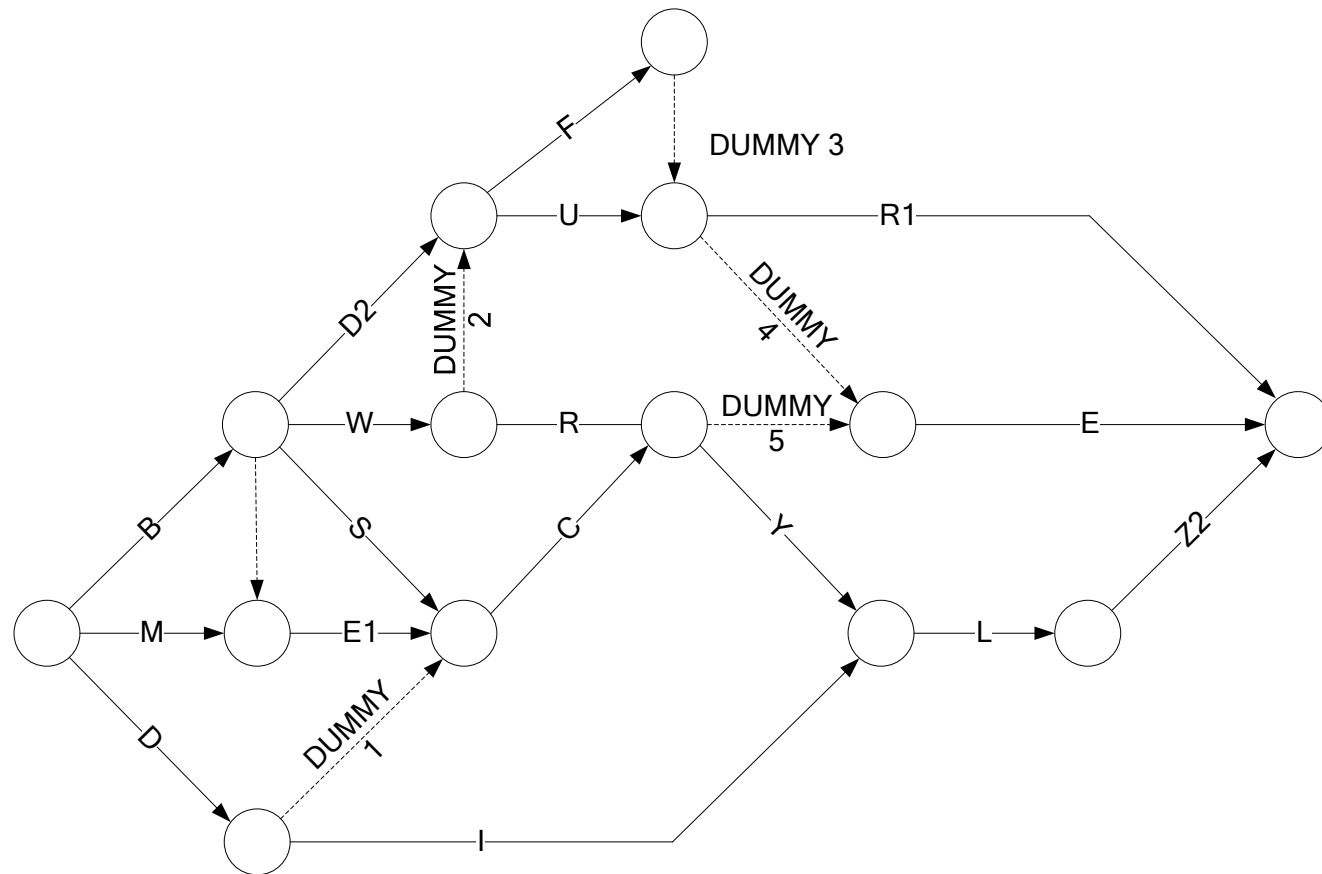
Solution 1.2.a



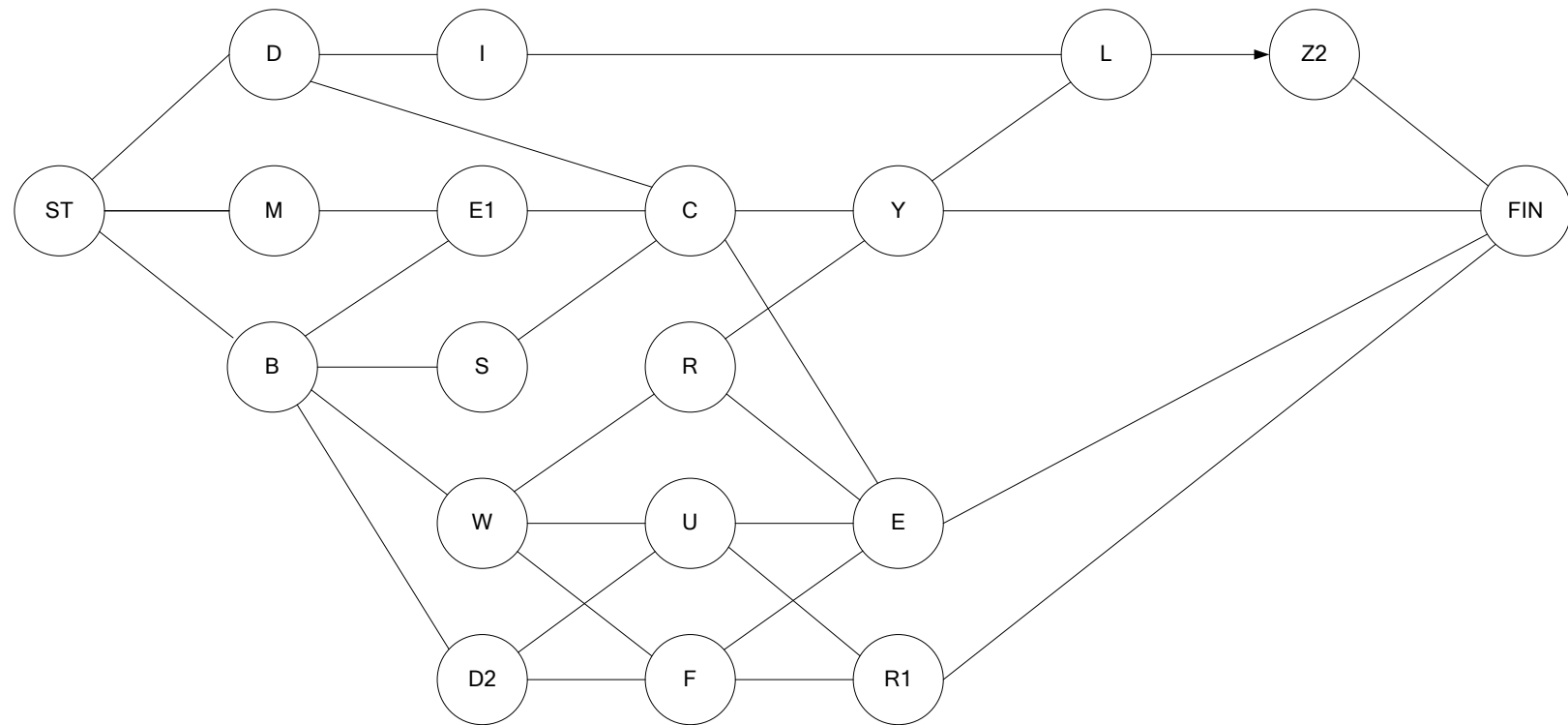
Solution 1.2.b



Solution 1.3.a



Solution 1.3.b

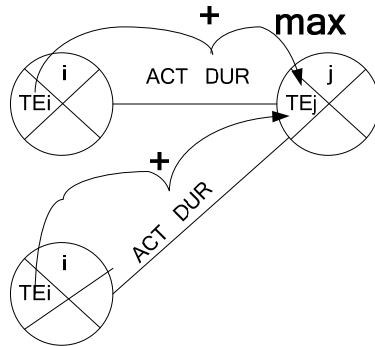


2. ACTIVITY-ON-ARROW SCHEDULING

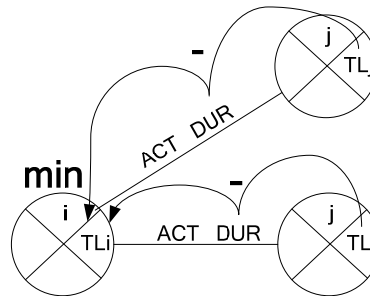
- EVENT APPROACH
- MISSING FLOATS

Calculation for activity-on-arrow networks

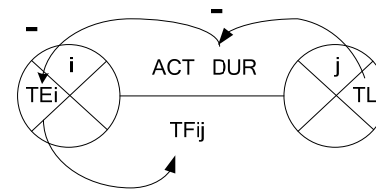
1) Calculate TE, Time Early
 $TE_j = \max(TE_i + DUR)$



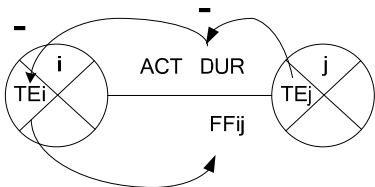
2) Calculate TL, Time Late
 $TL_i = \min(TL_j - DUR)$



3) Calculate TF, Total Float
 $TF_{ij} = TL_j - DUR - TE_i$

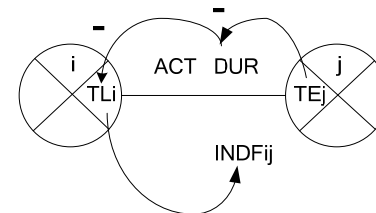


4) Calculate FF, Free Float
 $FF_{ij} = TE_j - DUR - TE_i$



5) Calculate INTF, Interfering Float
 $INTF_{ij} = TF_{ij} - FF_{ij} \text{ or } TL_j - TE_j$

6) Calculate INDF, Independent Float
 $INDF_{ij} = TE_j - DUR - TL_i$



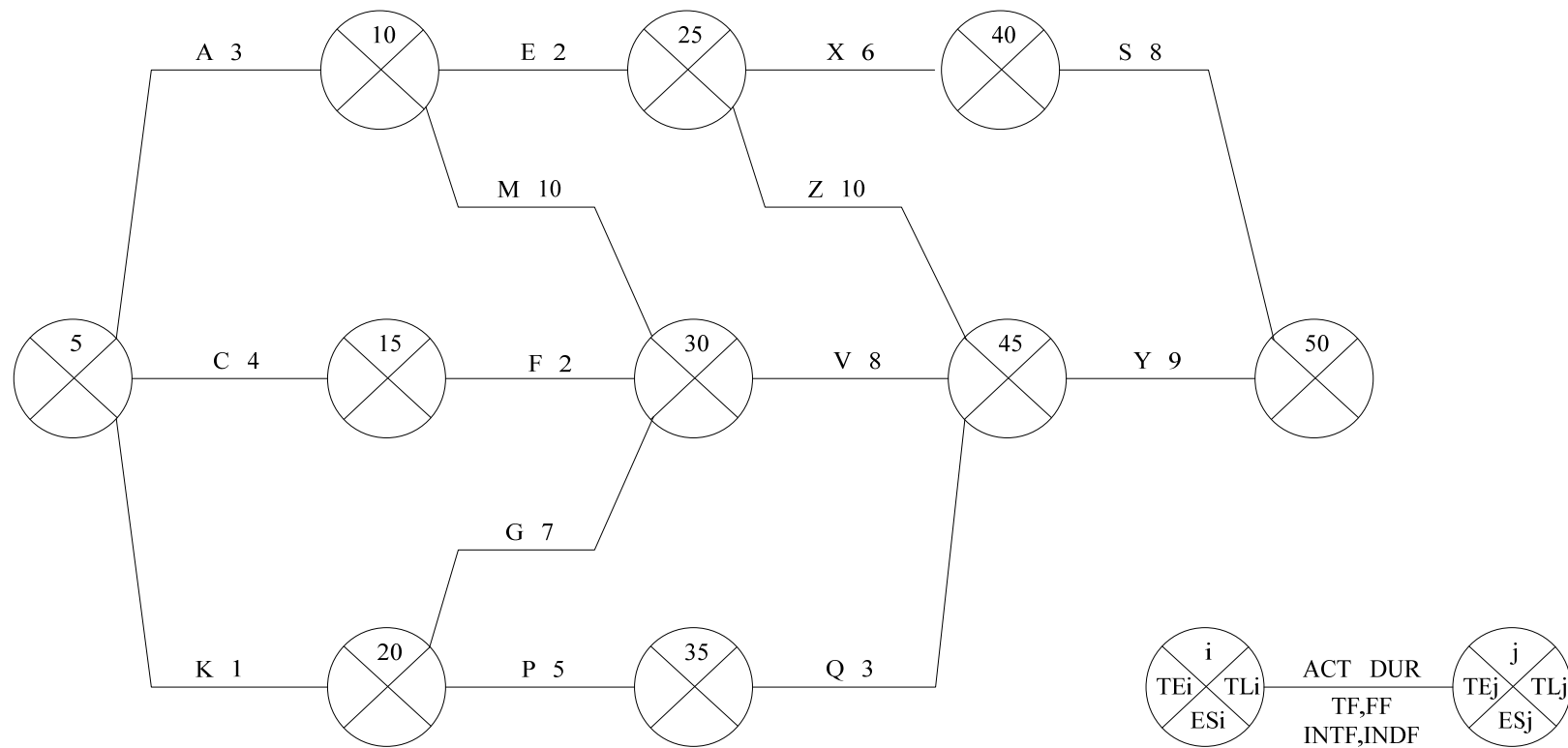
Note:

$TF \Rightarrow FF \geq INDF$ and $TF \geq INTF$.

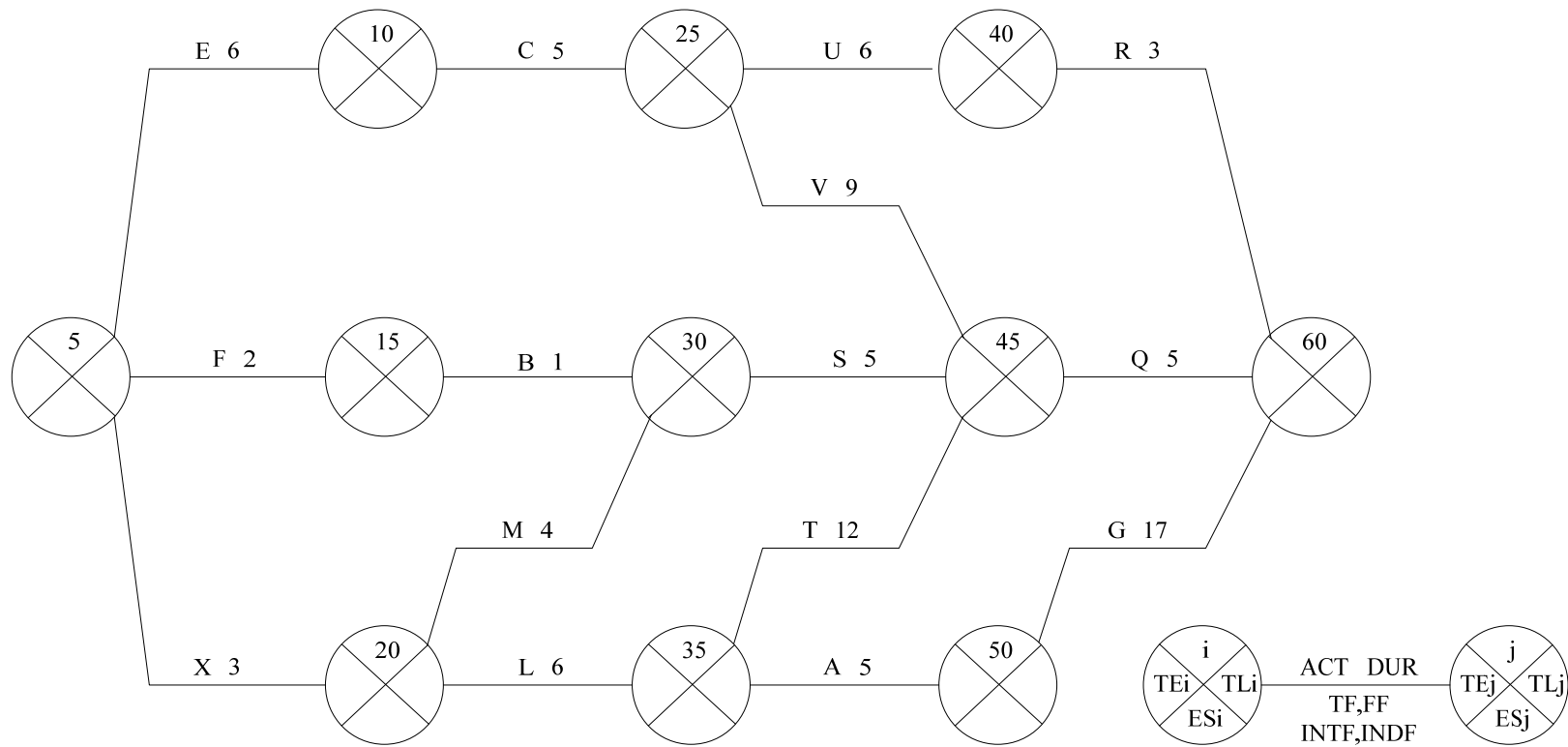
If $TF = 0$ then all the floats = 0.

If $FF = 0$ then $INDF = 0$. Remember that $INDF$ is a part of FF .

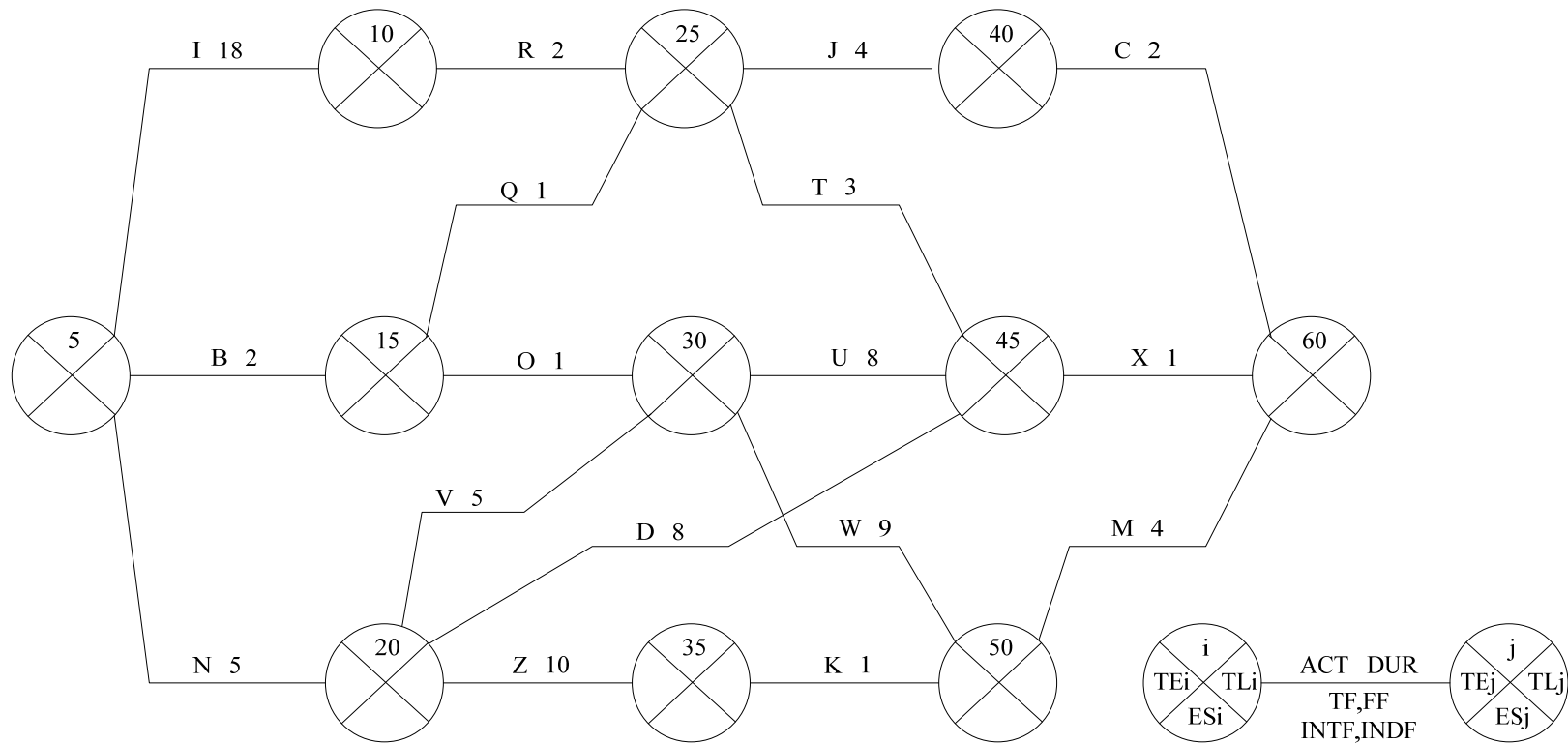
Problem 2.1 Calculate the schedule dates (TE and TL) and the four floats (TF, FF, INTF, and INDF).



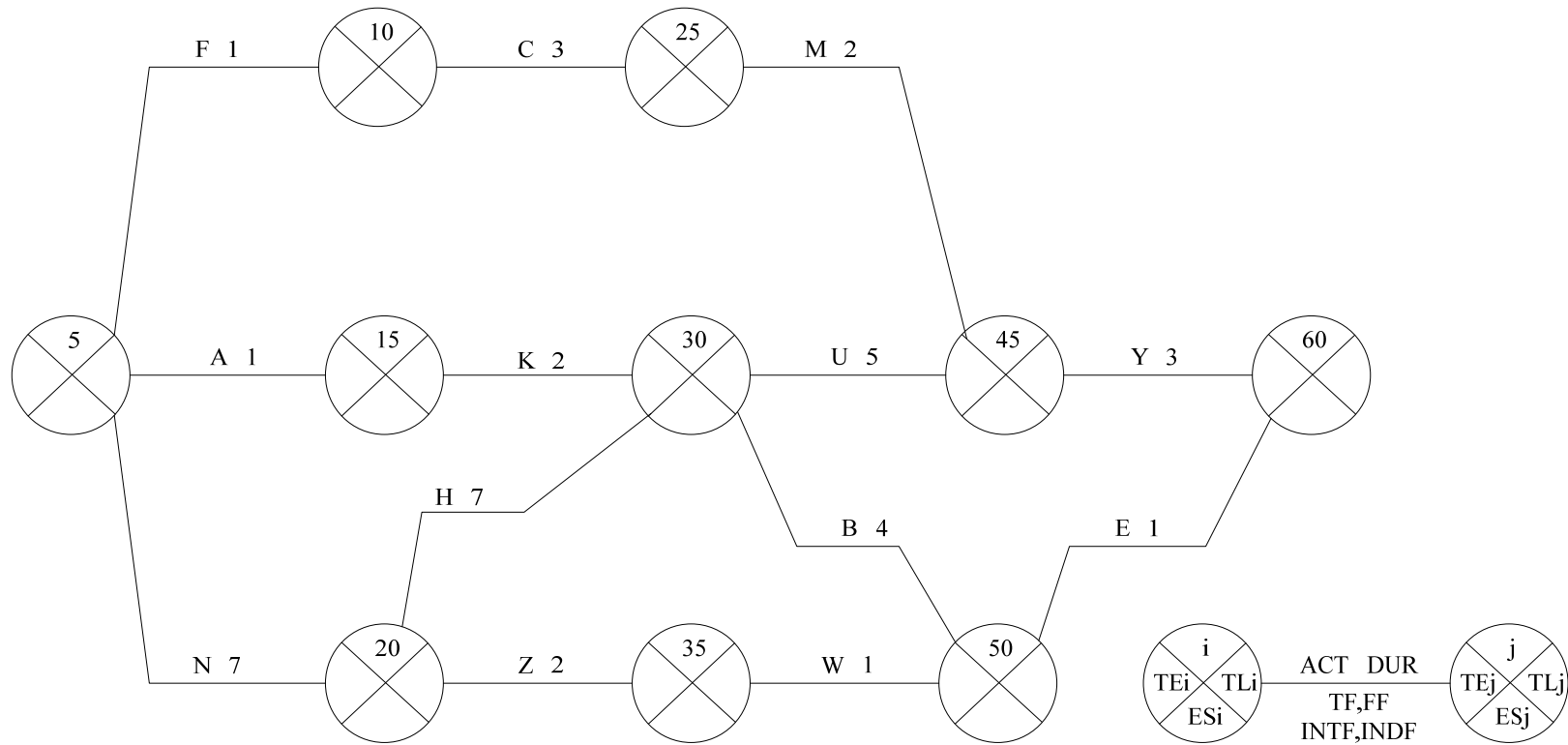
Problem 2.2 Calculate the schedule dates (TE and TL) and the four floats (TF, FF, INTF, and INDF).



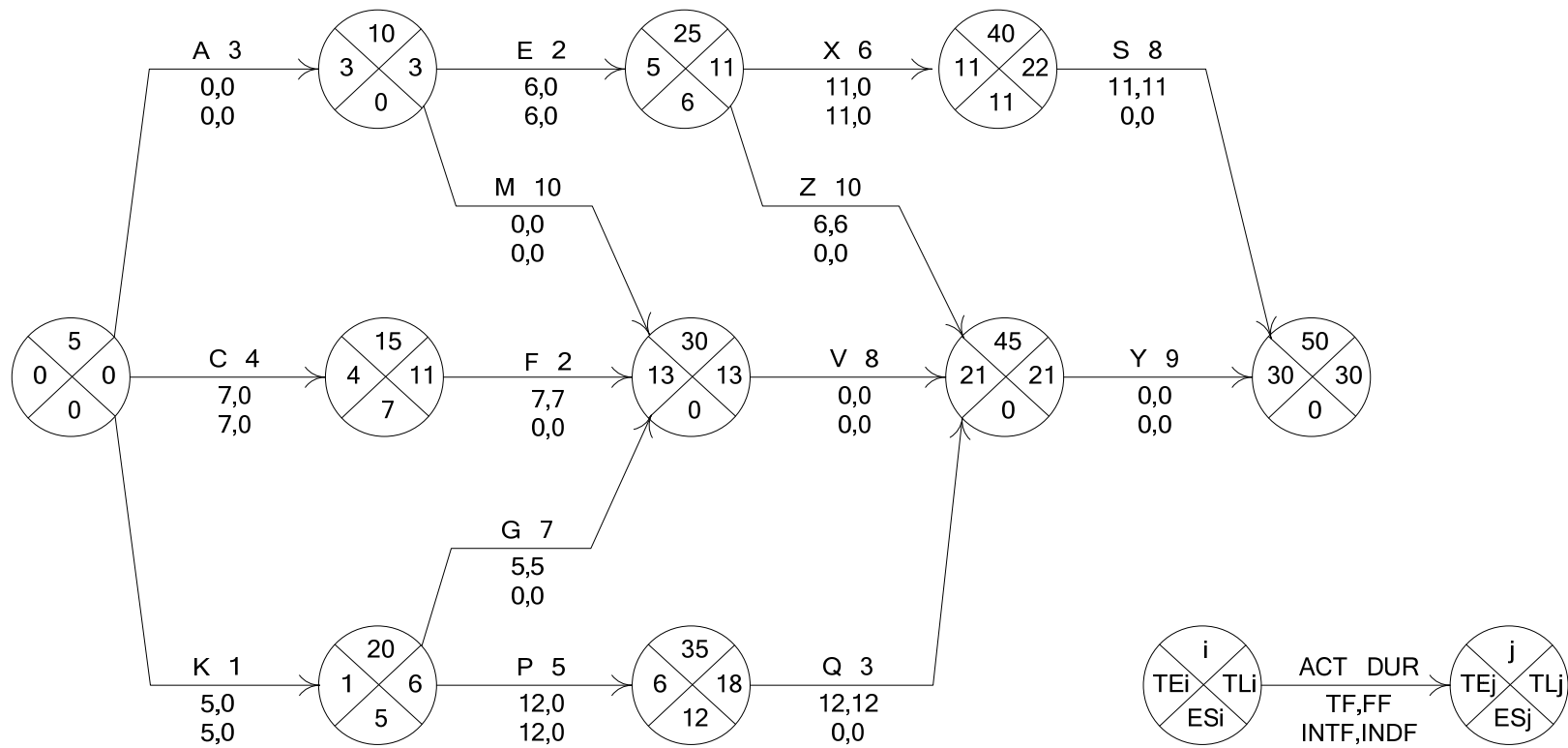
Problem 2.3 Calculate the schedule dates (TE and TL) and the four floats (TF, FF, INTF, and INDF).



Problem 2.4 Calculate the schedule dates (TE and TL) and the four floats (TF, FF, INTF, and INDF).



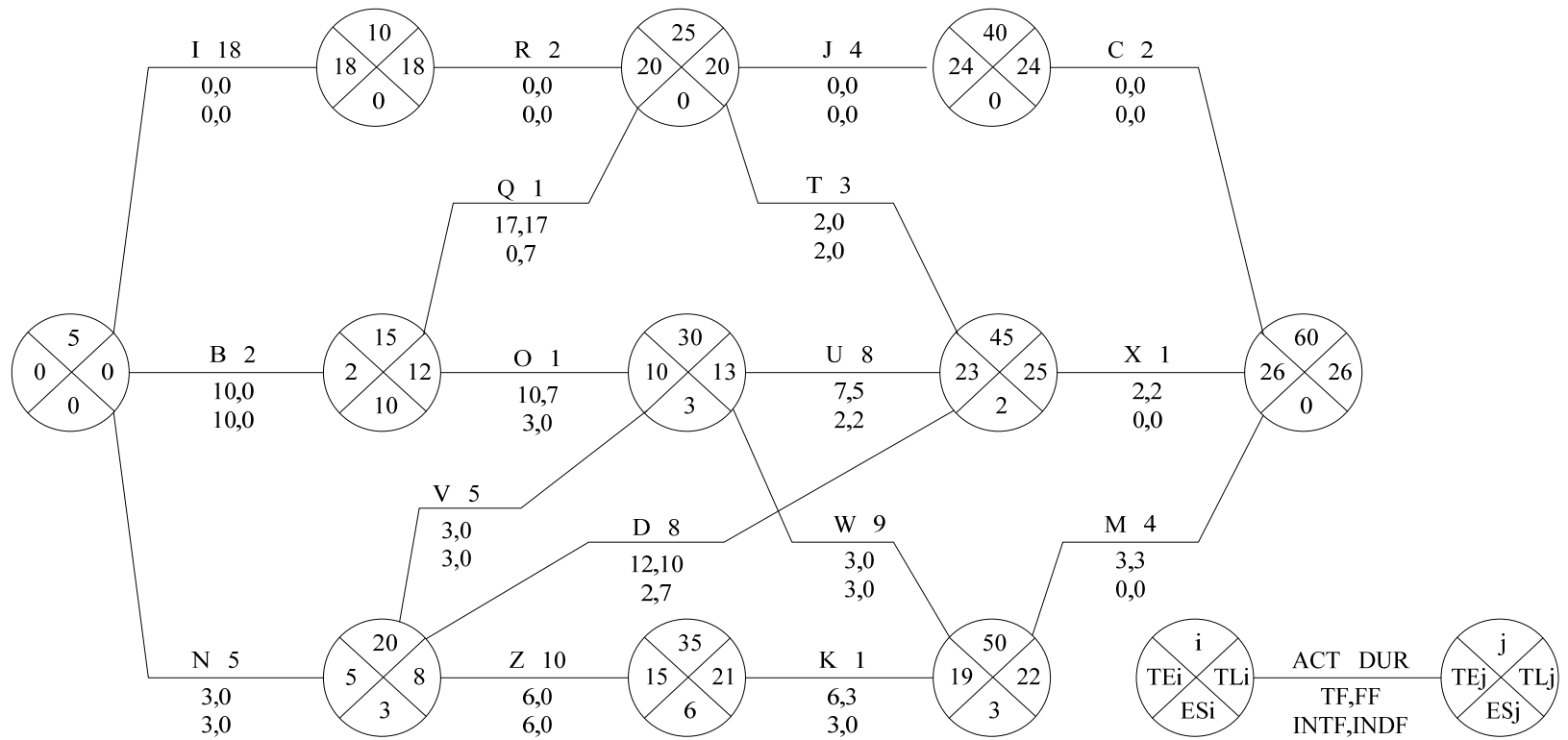
Solution 2.1



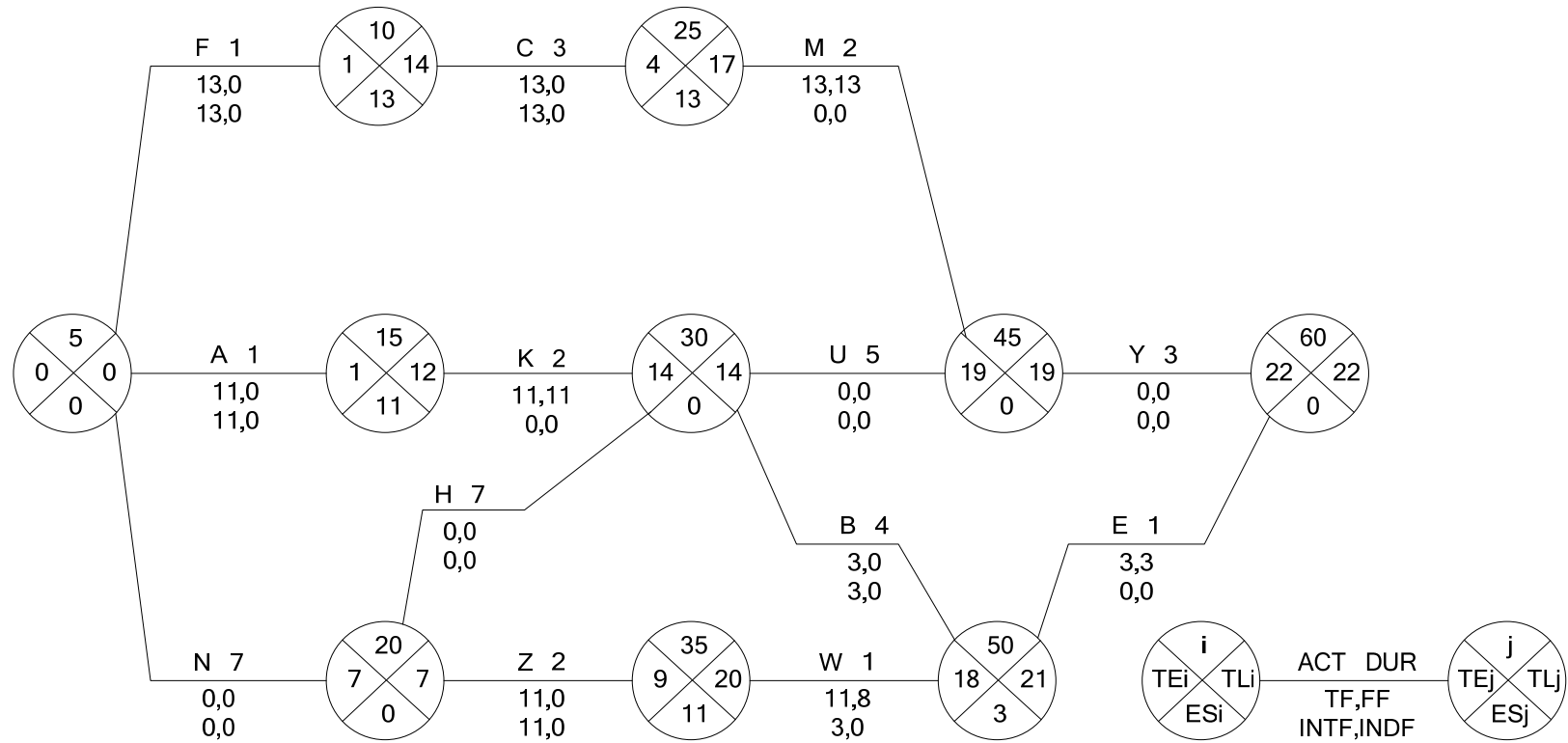
CEE536—Example Problems



Solution 2.3



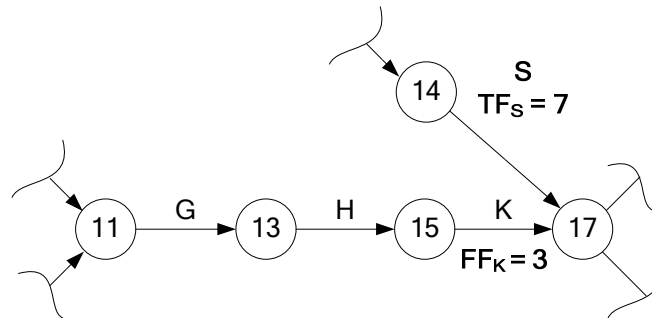
Solution 2.4



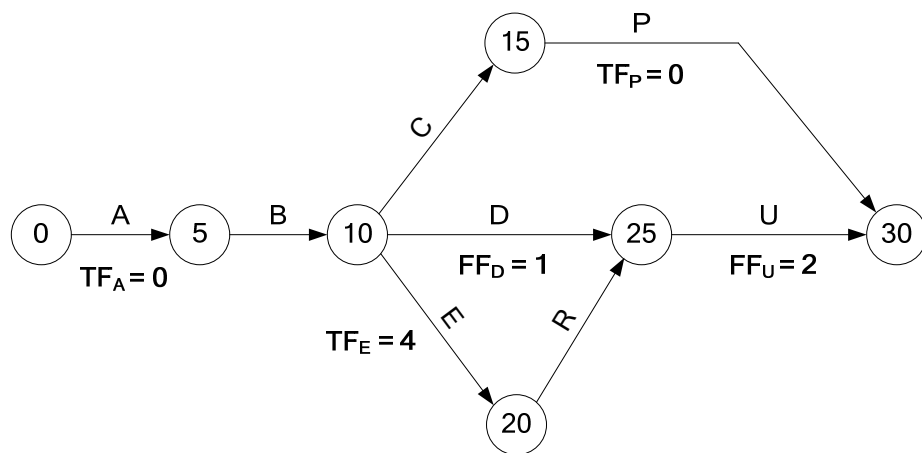
MISSING FLOATS

Calculate TF, FF, and INTF.

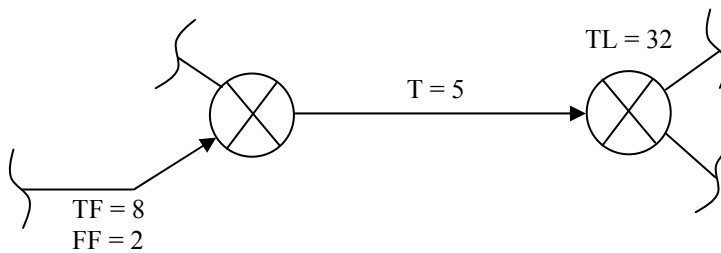
1.



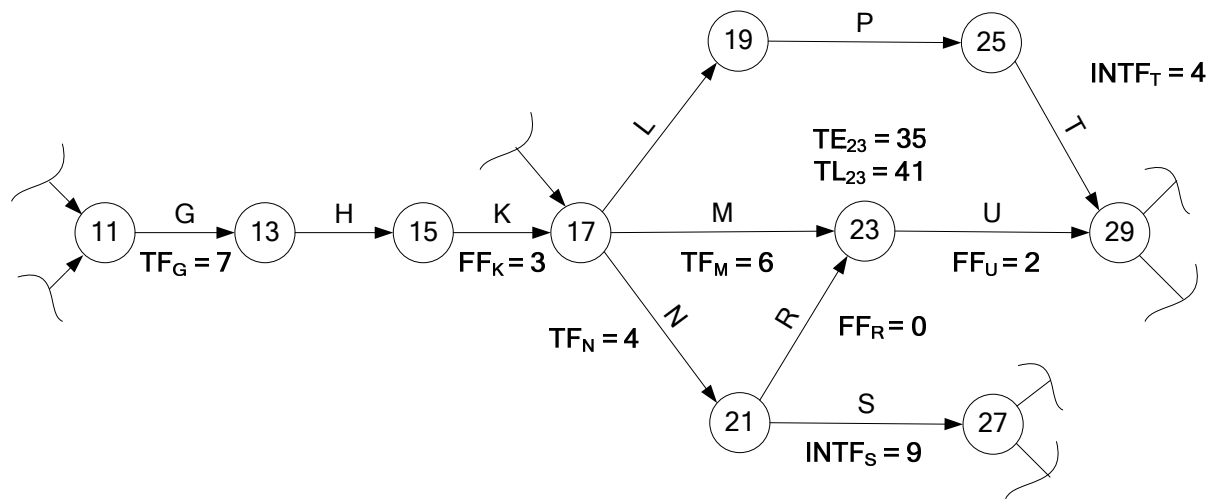
2.



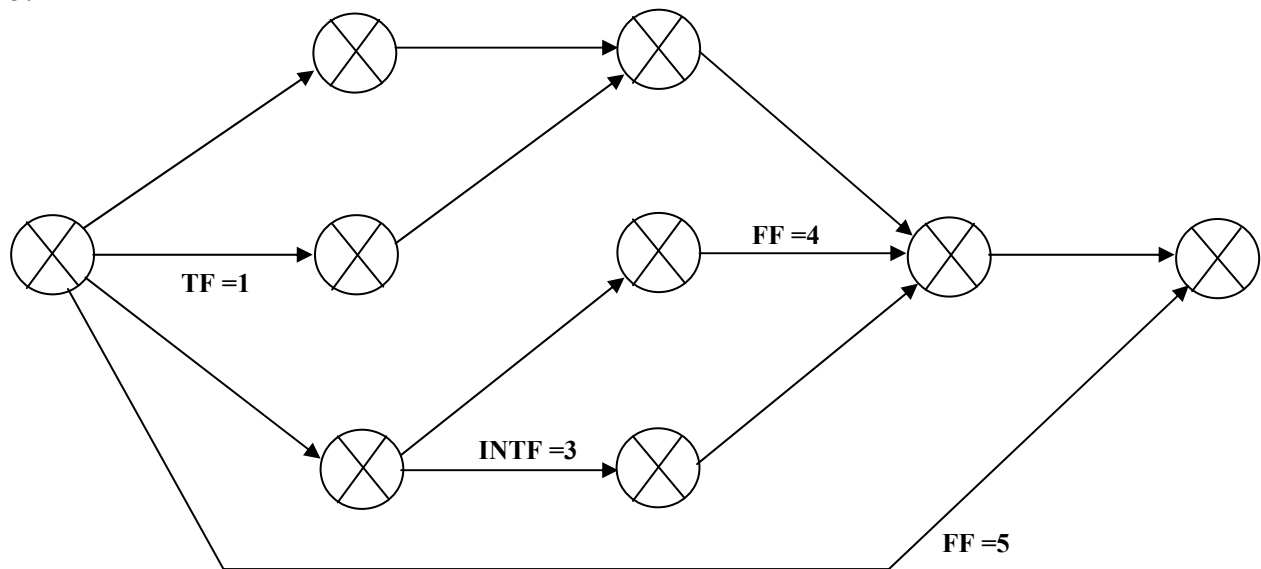
3.



4.



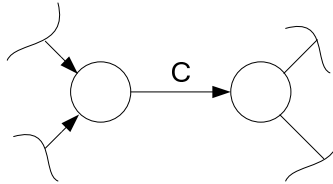
5.



FOUR MISSING FLOAT CONCEPTS (please check these concepts with the previous activity-on-arrow practice)

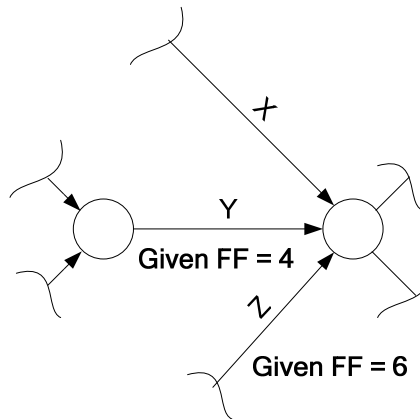
MS1. ZERO FREE FLOAT

If there is only one link goes into a node, its $FF = 0$ ”.



FF of activity C = 0

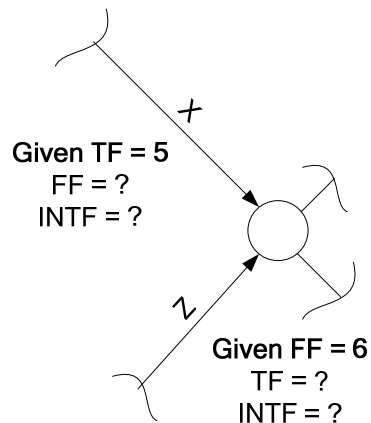
If there are many links go into the same node, at least one of them must have $FF = 0$.



FF of activity X = 0

MS2. SAME INTERFERE FLOAT

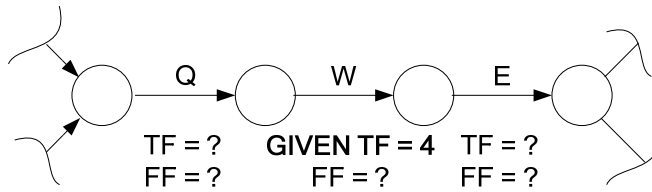
All the links that go into the same node have the same INTF.



According to MS1, FF of X = 0. Thus, $INTF$ of X = 5.
 According to MS2, $INTF$ of Z equals to $INTF$ of X, which is 5.
 Thus, TF of Z is 11.

MS3. ACTIVITY CHAIN'S TOTAL FLOAT

Total floats of activities on an activity chain are the same.

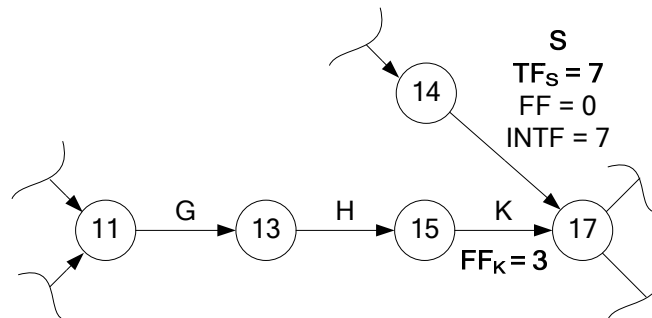


Activity Q, W, and E are activity chain. Thus,
TF of Q and E equal to TF of W according to MS3.
FF of Q, W, and E equal to 0 according to MS1.

Solution

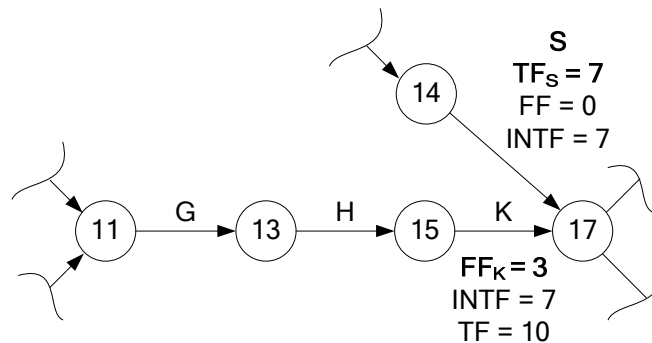
1.

1.1 From MS1, at node 17, since two links go into the same node and FF of K = 3, FF of S = 0. Thus, INTF of S = 7.



1.2 From MS2, links go into the same node have the same INTF.

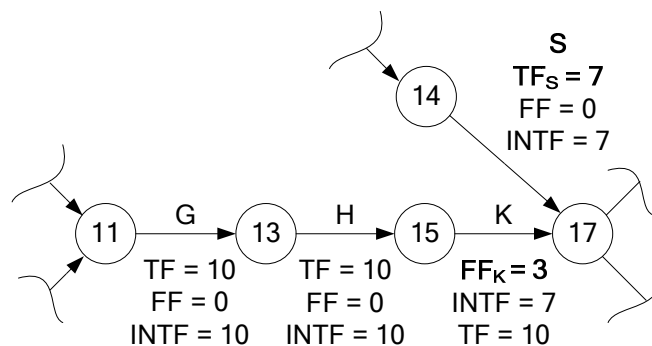
Thus, INTF of K = 7, and TF of K = 10.



1.3 From MS3, activities in the activity chain have the same TF.

Thus, TF of K, G, and H are 10.

1.4 From MS1, FF of G and H = 0. INTF of G and H are 10.



2.

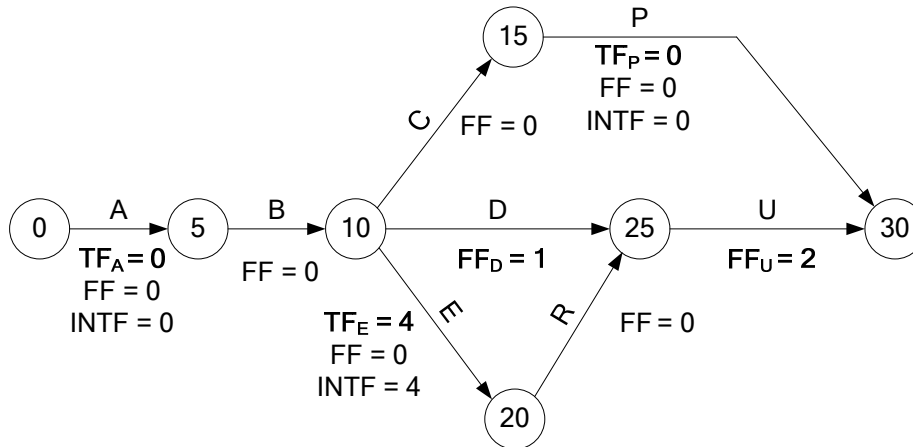
2.1) MS1 (one link goes into one node, FF of the link = 0)

FF of A, B, C, and E = 0.

So, INTF of A and E = 0 and 4 respectively.

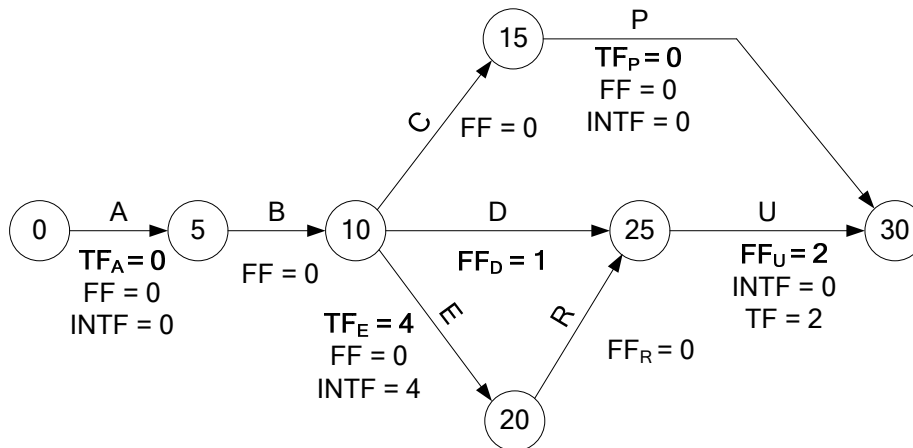
MS1 (many links going to the same node, one of them must have zero FF)

FF of P = 0. Thus, INTF of P = 0. And also FF of R.



2.2) MS2 (many links going into the same have the same INTF)

INTF of U = INTF of P = 0. So, TF of U = 2.



2.3) MS3 (activities on activity chain have the same TF)

TF of B = TF of A.

TF of R = TF of E. Then calculate INTF of R, which will give us INTF of D according MS2.

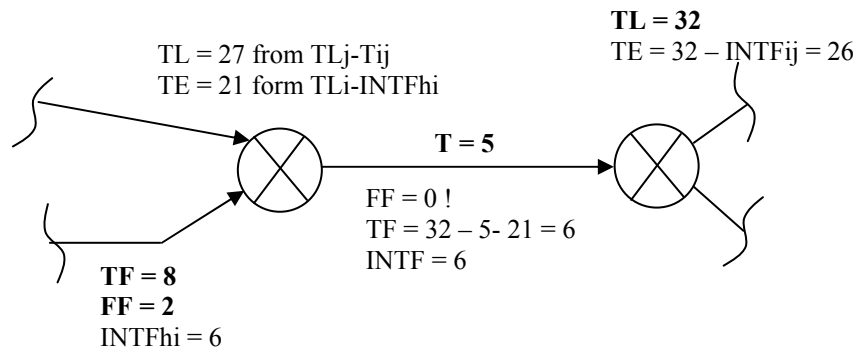
Thus, TF of D can be calculated. 😊

TF of C = TF of P.

Note: up to this point, you should be able to get all the TF, FF, and also INTF. It should also be mentioned that the given TF of A and P are not necessary. WHY???

3. **One link goes into one node** $\rightarrow$ $FF = 0$

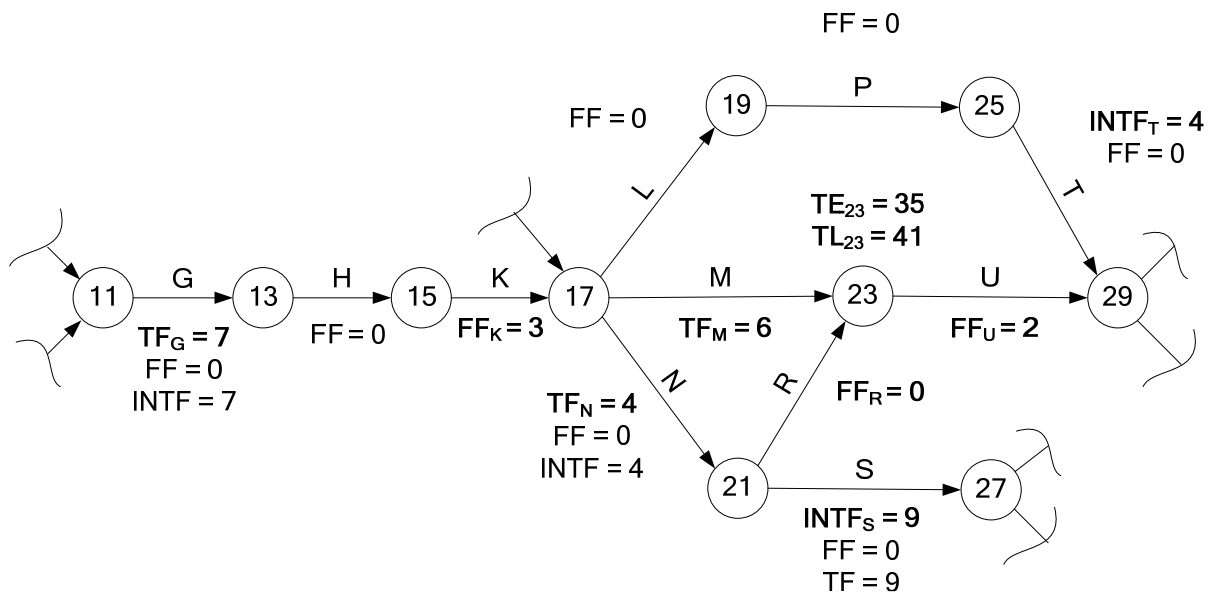
Or **Many links go into the same node** $\rightarrow$ **at least one of the links must have $FF = 0$**



4.

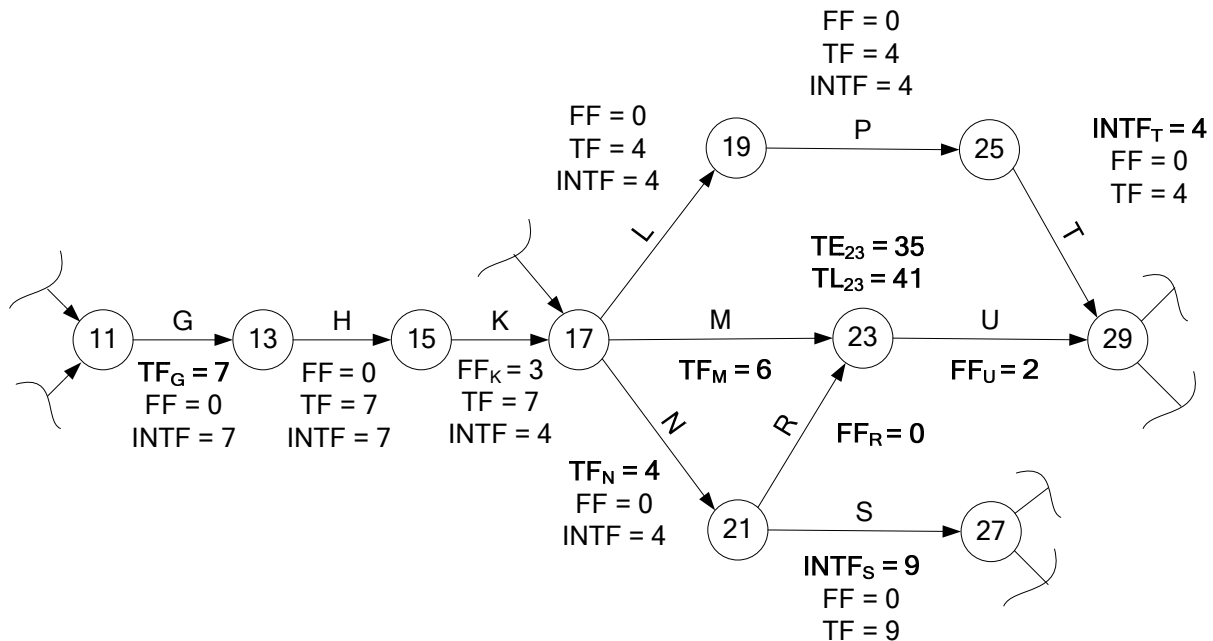
4.1) “**Only one link goes into one node**”, we should be able to spot out activities whose $FF = 0$, which are activities G, H, N, L, P, and S.

4.2) “**Many links go into the same node, at least one of them must have $FF = 0$** ”. Thus, FF of $T = 0$



4.3) “Total float on activities on an activity chain have the same TF”

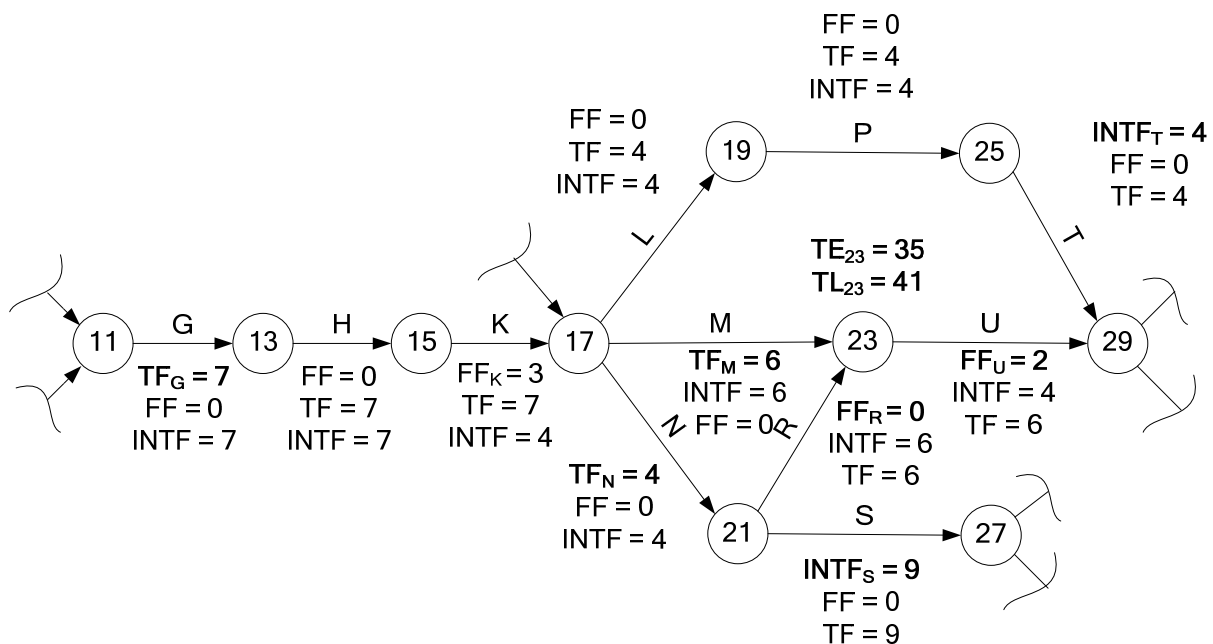
Thus, we should be able to get TF of H and K (from Chain G-H-K), L and P (from Chain L-P-T).



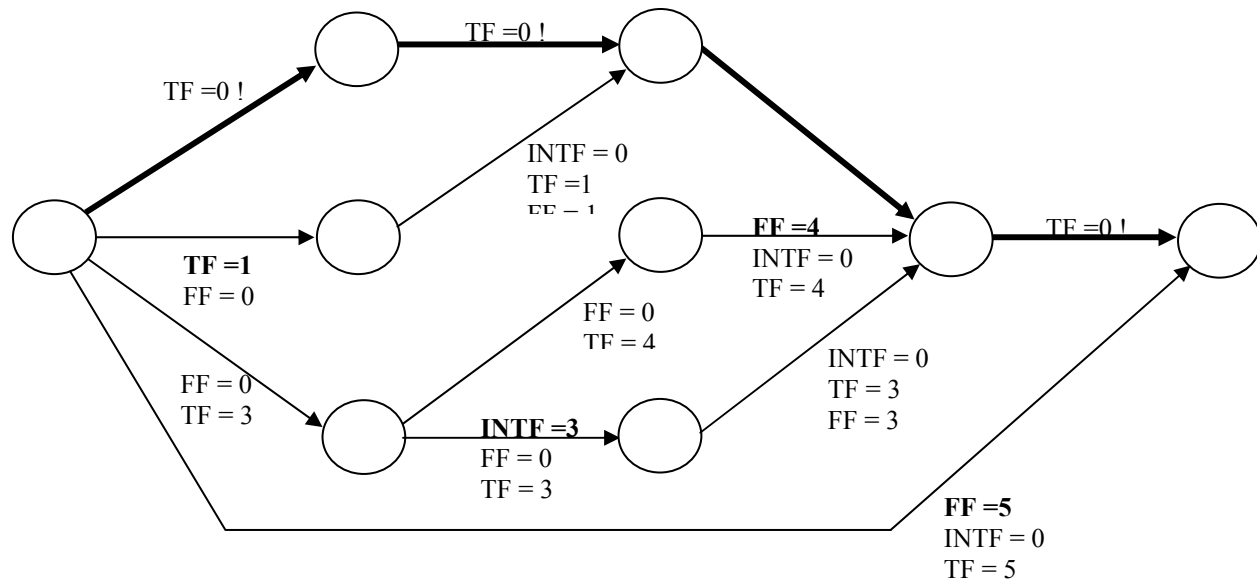
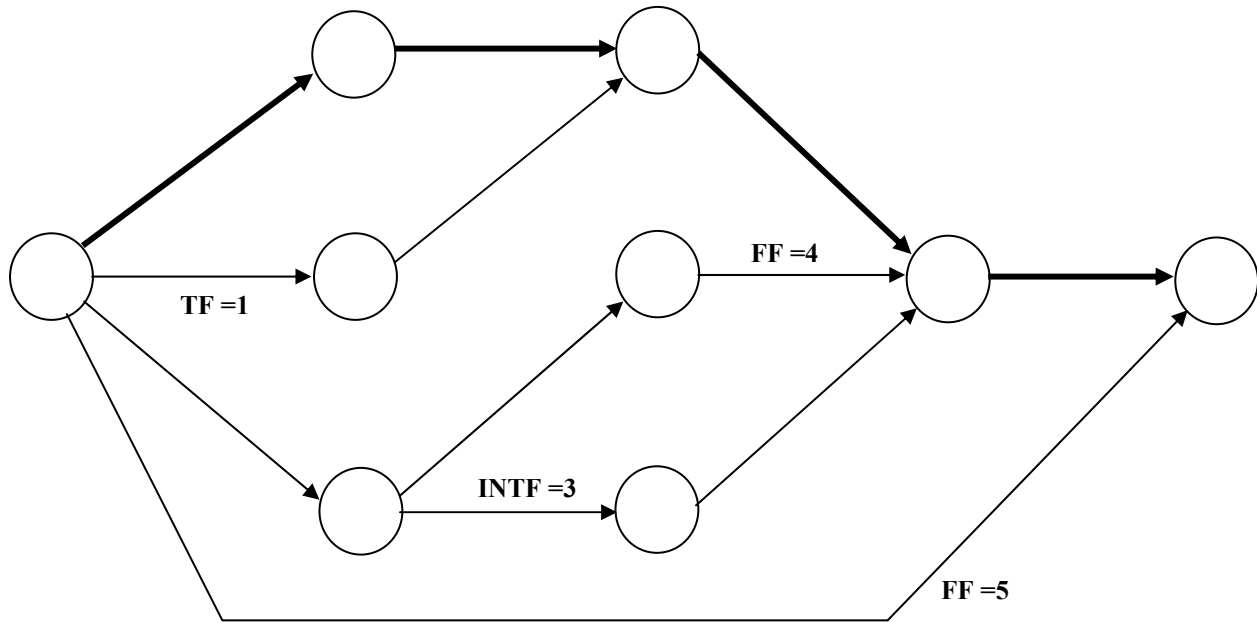
4.4) “Activities going to the same node have the same INTF”

Thus, we can calculate INTF of M and R (going to node 23), and U (going to node 29).

Note: INTF of M and R is $TL_{23} - TE_{23}$ (you should remember this by now)



5. It is crucial to identify critical path, and knowing that any links going to the **same node** have the **same INTF**.



3. ACTIVITY-ON-NODE SCHEDULING

Problem 3.1

- A. Construct a precedence diagram.
- B. On the diagram, compute the four schedule dates (ESD, EFD, LSD, LFD) and the four floats (TF, FF, INTF, and IDF) for each activity, and the lag for each link.
- C. Identify the critical path

No	ACT	DUR	PREDECESSORS
5	B	5	
10	M	4	B
15	N	9	B
20	Q	15	B
25	A	1	M,N
30	F	4	N,Q
35	X	9	Q
40	C	9	Q
45	Y	9	A,F,X
50	S	6	F
55	J	5	X,F
60	T	10	C
65	V	5	Y,S
70	U	10	V,T,J

Problem 3.2

- A. Construct a precedence diagram.
- B. On the diagram, compute the four schedule dates (ESD, EFD, LSD, LFD) and the four floats (TF, FF, INTF, and IDF) for each activity, and the lag for each link.
- C. Identify the critical path

No	ACT	DUR	PREDECESSORS
5	A	1	
10	B	8	A
15	C	4	A
20	P	7	A
25	L	2	B
30	M	4	C
35	Q	4	P,C
40	N	9	P
45	Y	5	L,Q
50	F	10	M
55	J	2	Q
60	S	2	N
65	V	5	Y,F,J
70	Q1	1	V,S

Problem 3.3

- A. Construct a precedence diagram.
- B. On the diagram, compute the four schedule dates (ESD, EFD, LSD, LFD) and the four floats (TF, FF, INTF, and IDF) for each activity, and the lag for each link.
- C. Identify the critical path

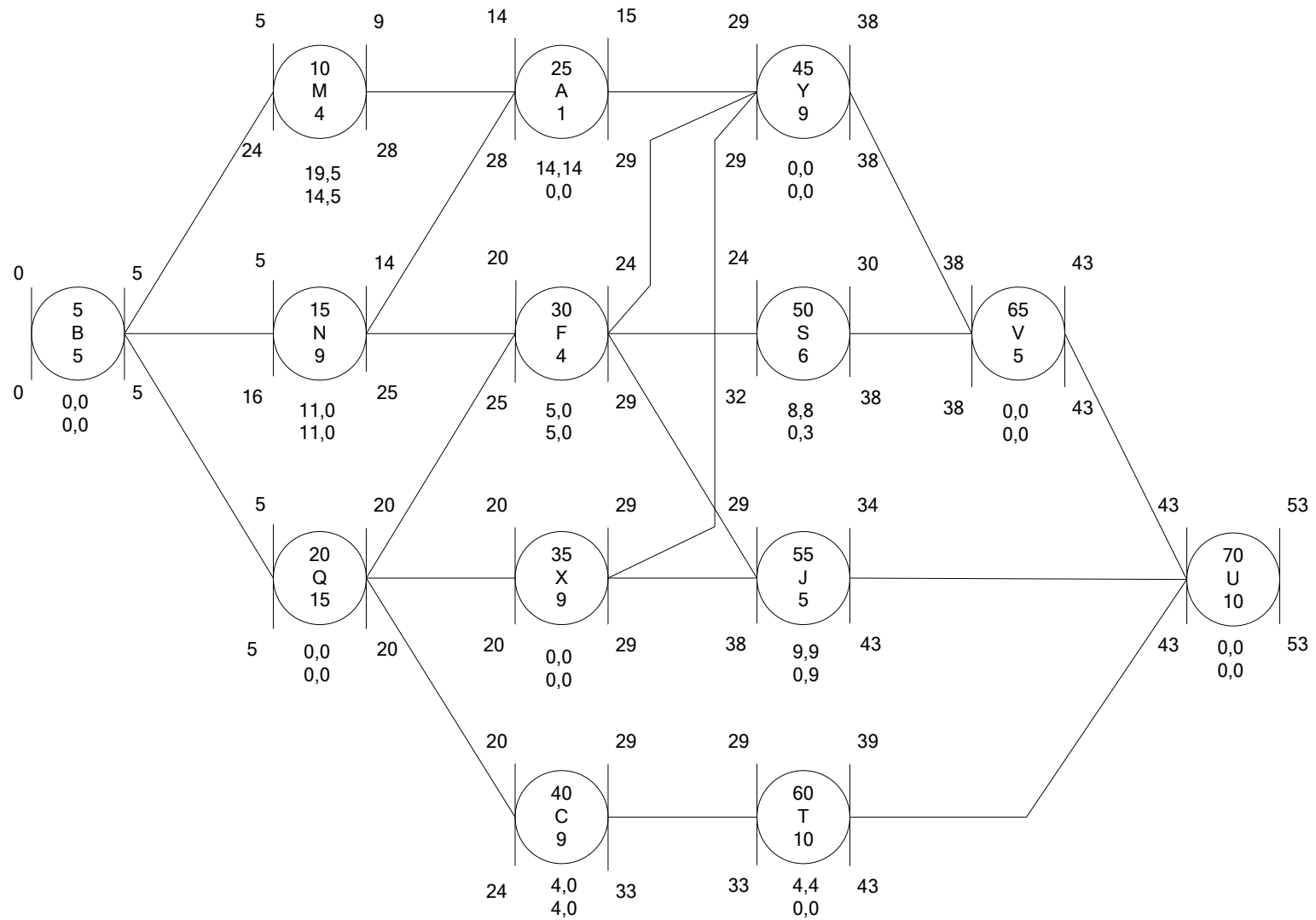
No	ACT	DUR	PREDECESSORS
5	B	5	
10	M	4	B
15	N	9	B
20	X	15	B
25	A	5	M,N
30	F	6	N,X
35	Q	2	X
40	C	4	X
45	Y	10	A
50	S	10	F,A
55	R	2	Q,F
60	T	5	C,Q
65	K	7	Y,S,R
70	U	3	K,T

Problem 3.4

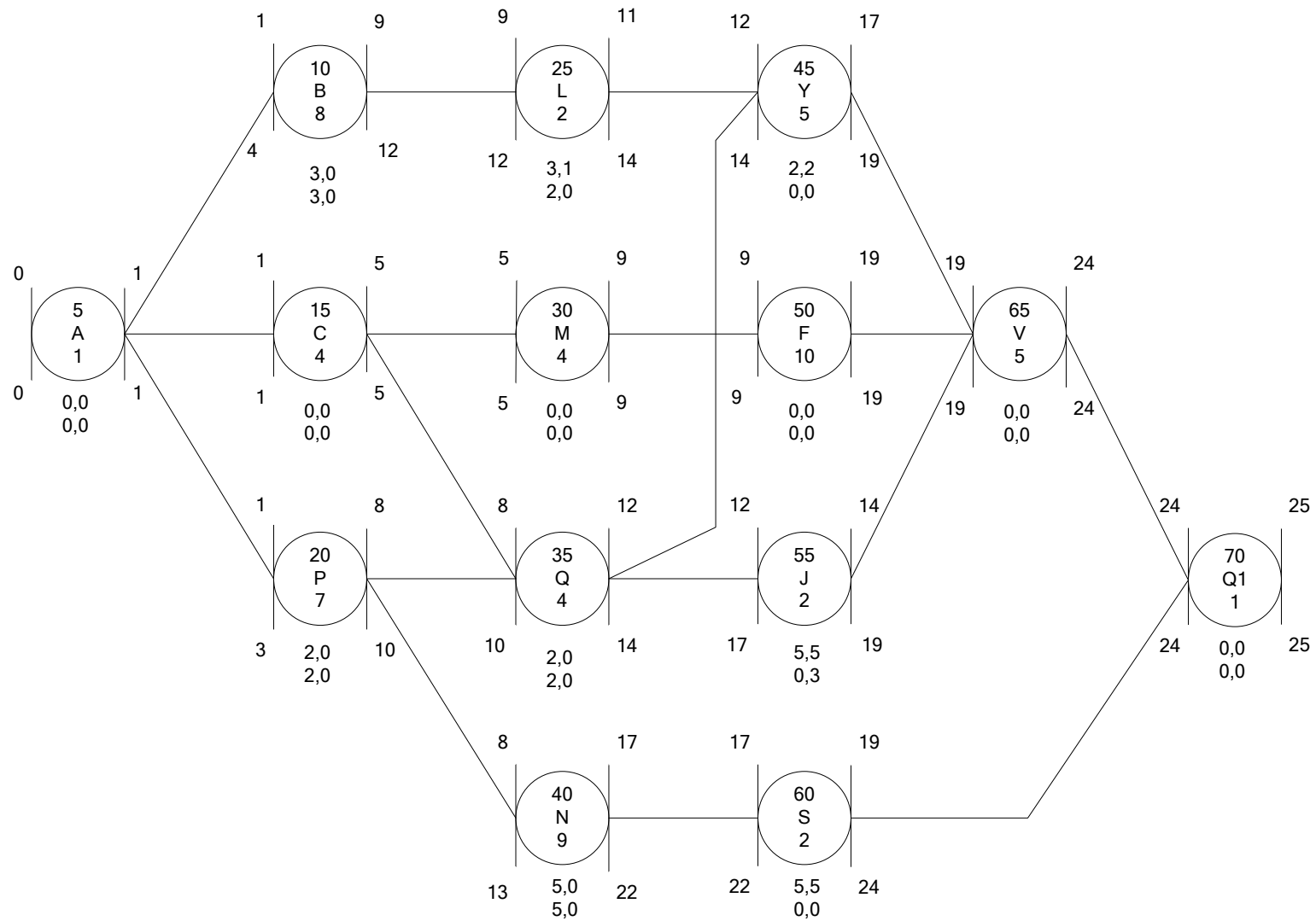
- A. Construct a precedence diagram.
- B. On the diagram, compute the four schedule dates (ESD, EFD, LSD, LFD) and the four floats (TF, FF, INTF, and IDF) for each activity, and the lag for each link.
- C. Identify the critical path

No	ACT	DUR	PREDECESSORS
5	A	9	
10	B	1	A
15	C	10	A
20	P	10	A
25	L	2	B,C
30	F	10	C,P
35	Q	8	P
40	N	6	P
45	Y	7	L
50	T	4	F,L
55	R	9	F,Q
60	S	1	N,Q
65	V	10	Y,T,R
70	U	1	V,S

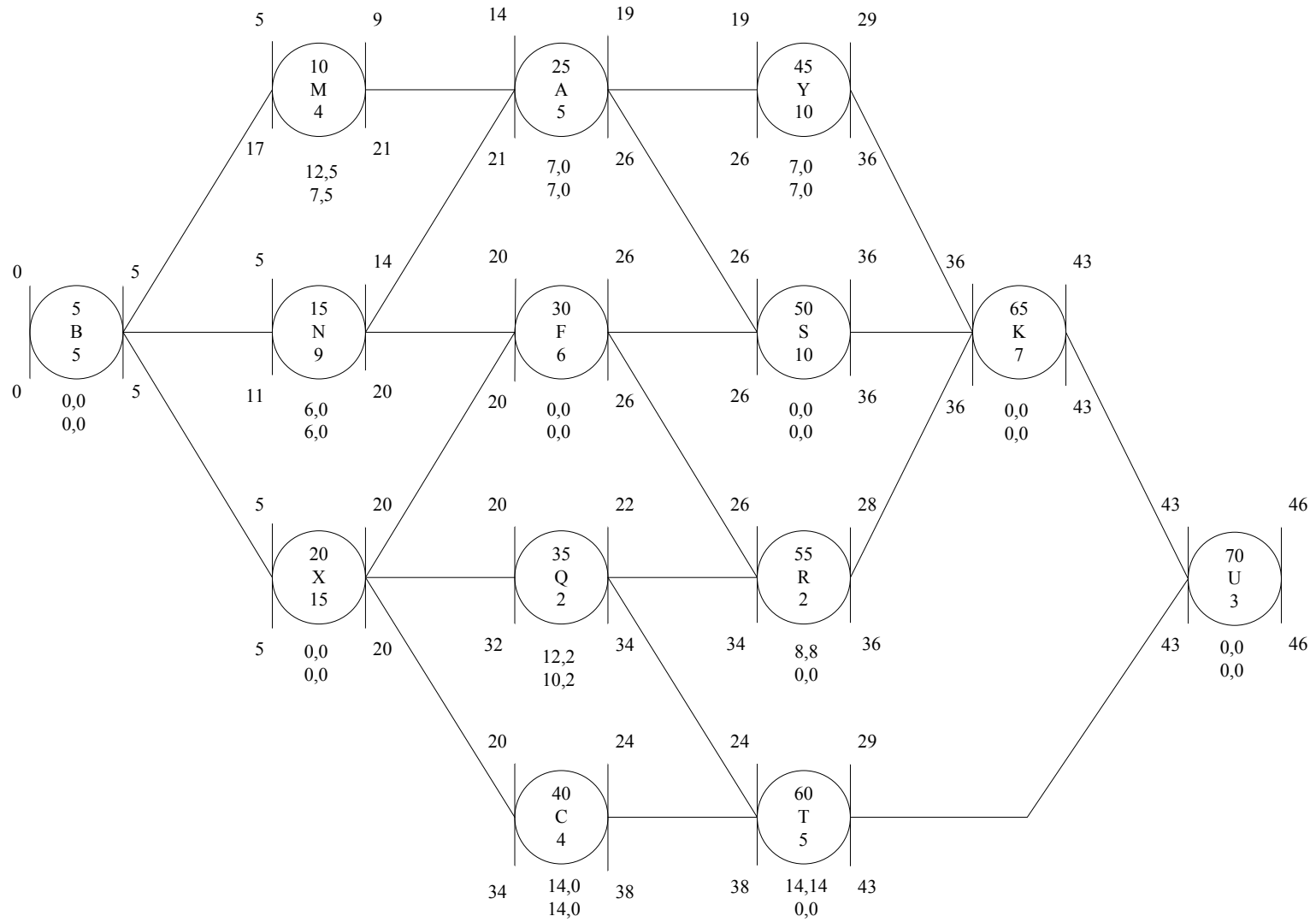
Solution 3.1



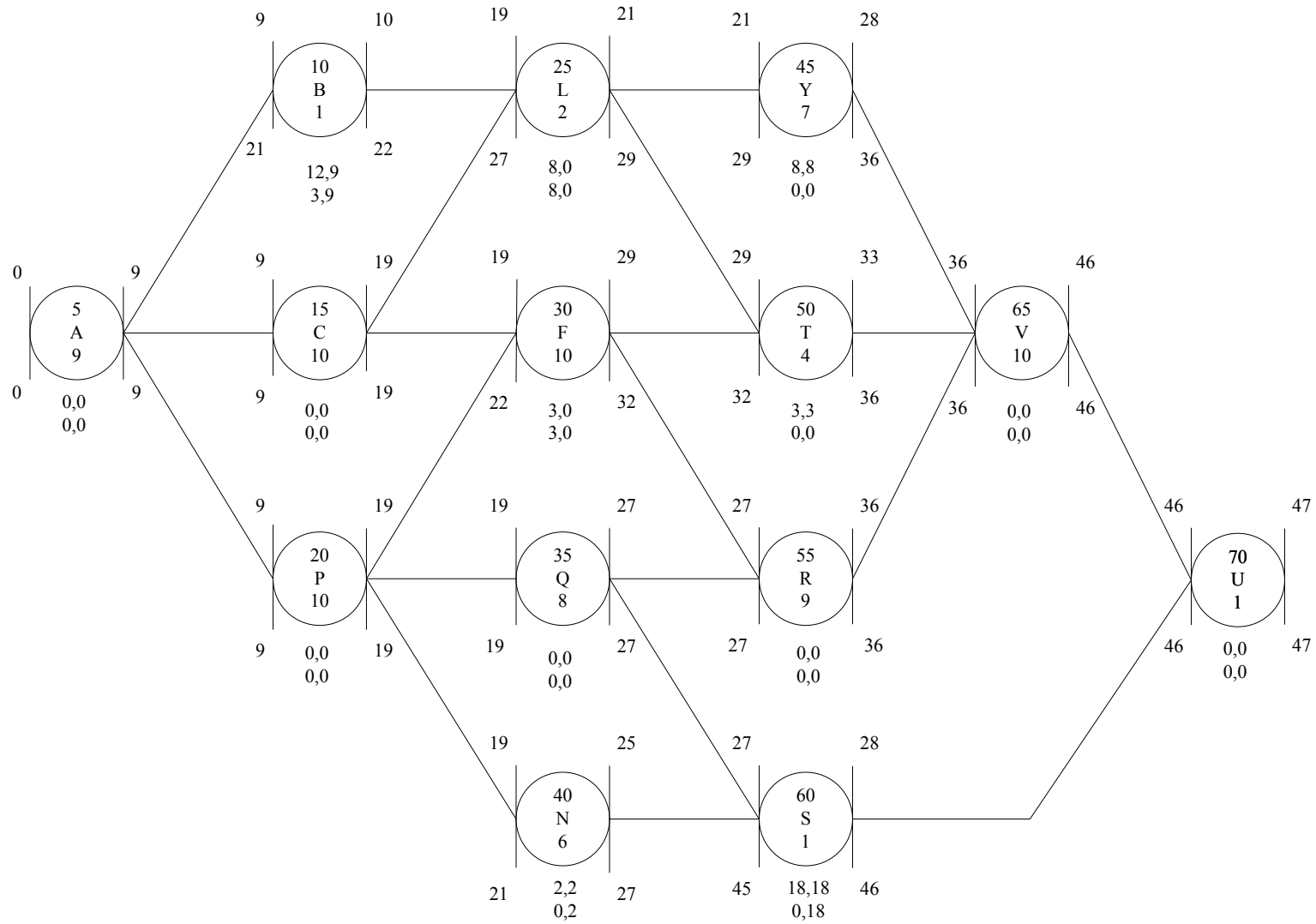
Solution 3.2



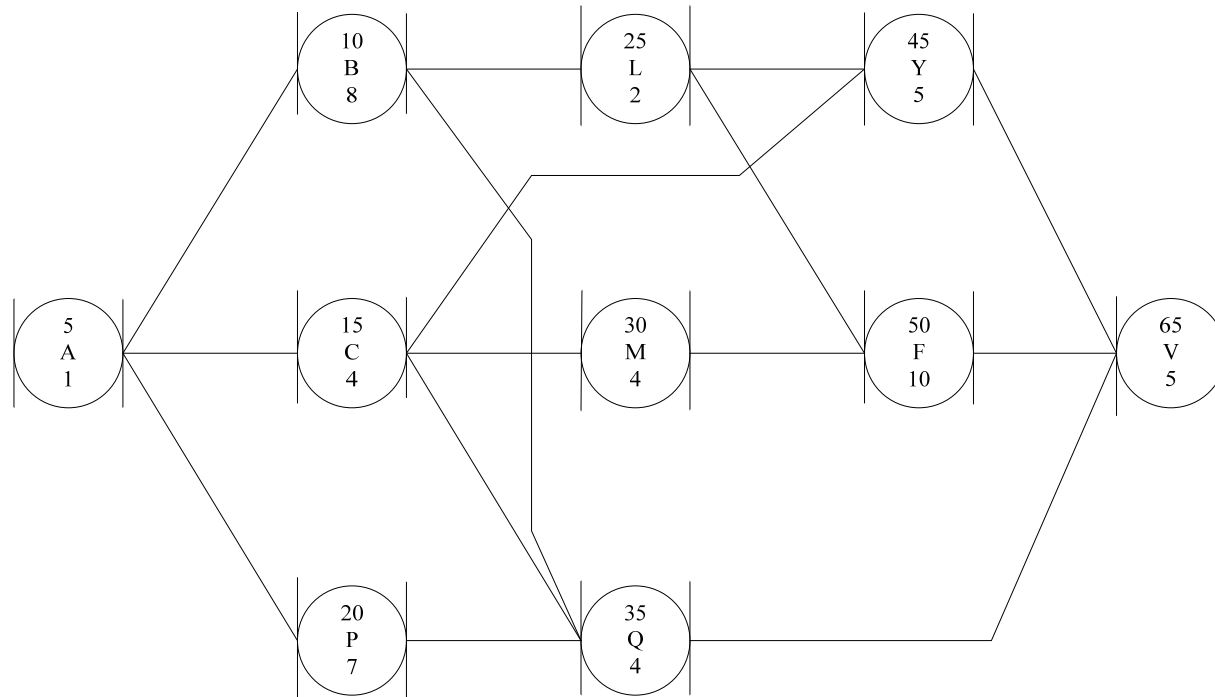
Solution 3.3



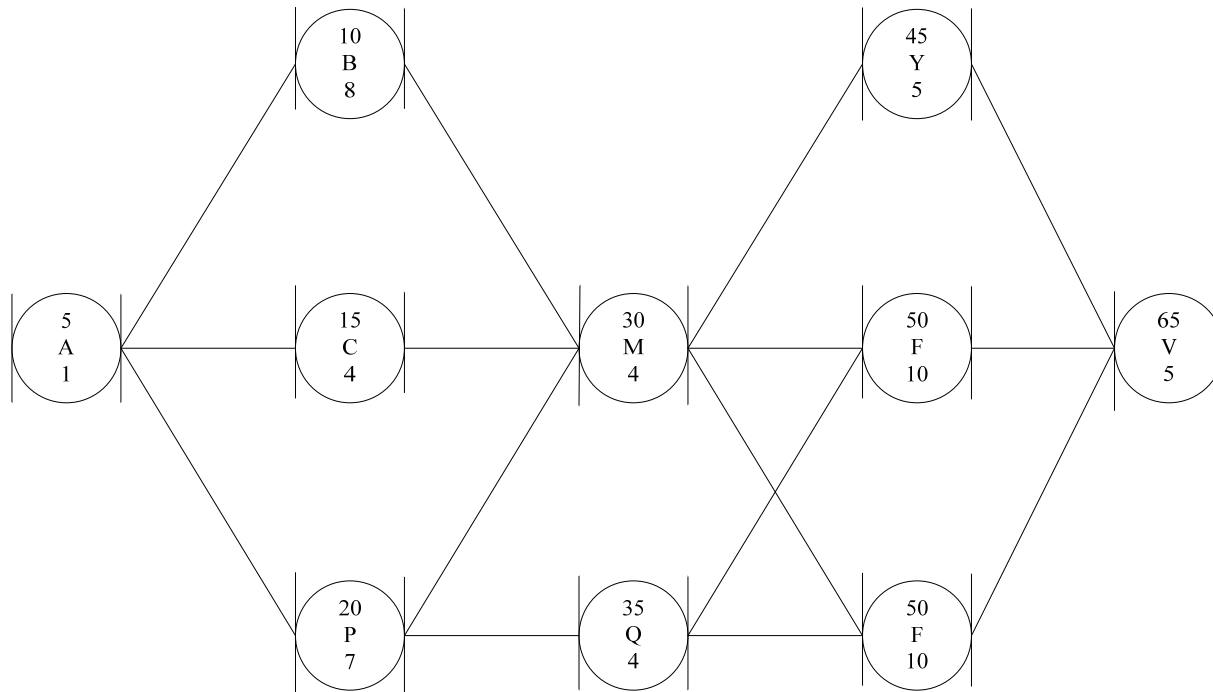
Solution 3.4



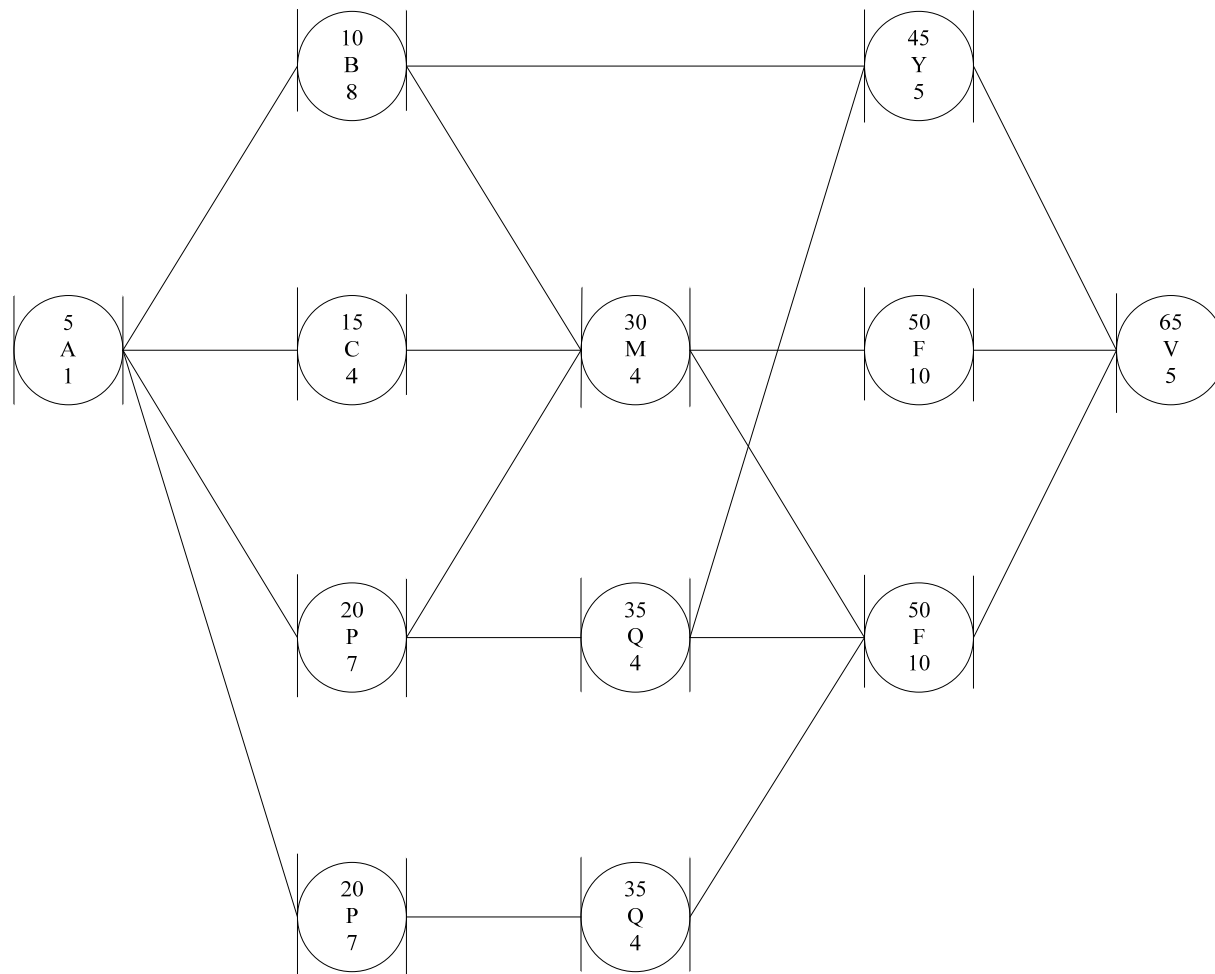
Problem 3.5



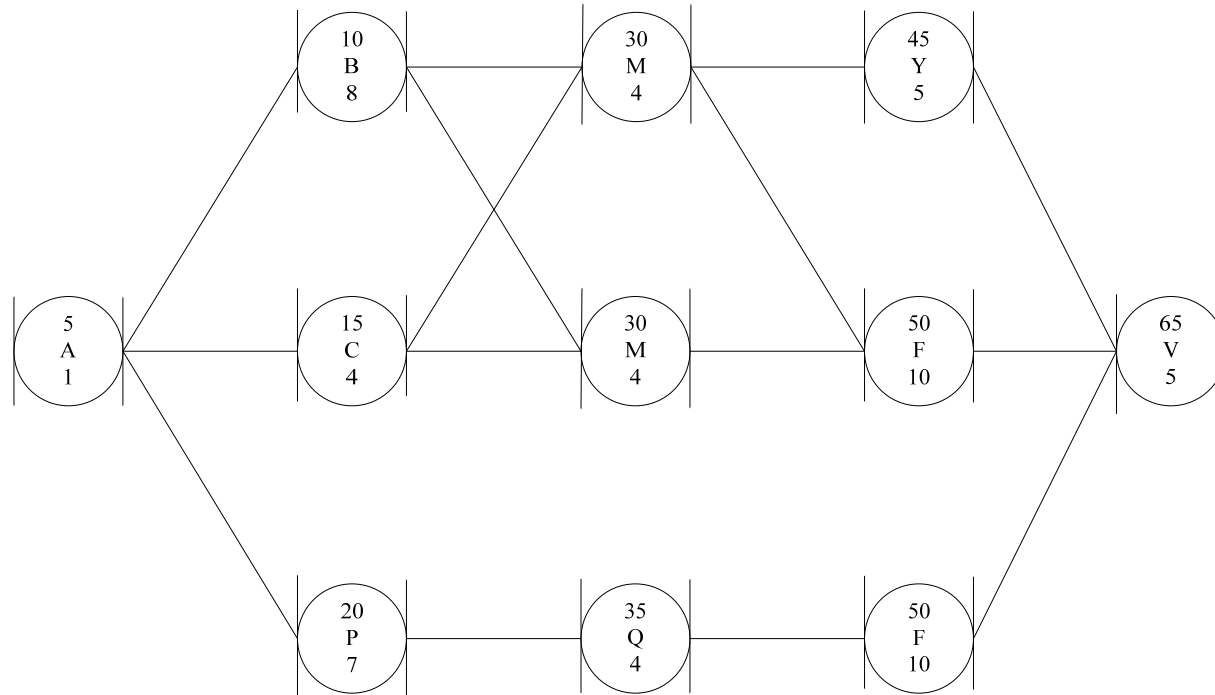
Problem 3.6



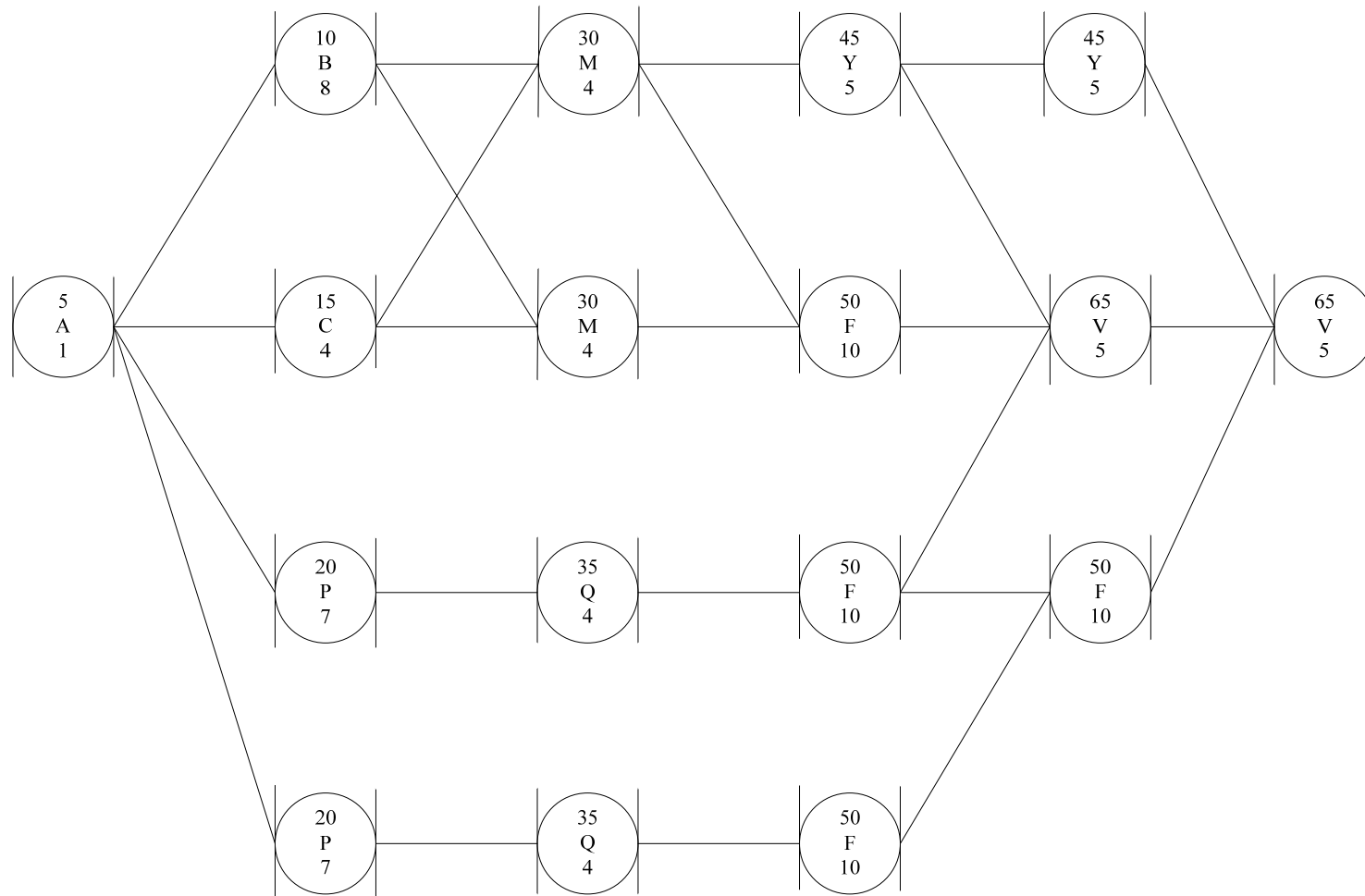
Problem 3.7



Problem 3.8



Problem 3.9 (CalActivityOnNode)



4. SCHEDULING USING A LINK MATRIX

Problem 4.1 Using matrix to calculate TF and FF

NO	ACT	DUR	PREDECESSORS
5	A	4	
10	B	5	A
15	C	3	A
20	Q	8	A
25	M	1	B,C
30	L	8	C,B,Q
35	X	2	Q,C
40	N	7	Q
45	Z	8	M,L
50	S	8	L,M
55	J	6	X,L
60	T	4	N
65	V	10	Z,S,J
70	Q1	6	V,T

Problem 4.2 Using matrix to calculate TF and FF

NO	ACT	DUR	PREDECESSORS
5	A	7	
10	B	5	A
15	C	7	A
20	P	5	A
25	M	1	B,C
30	L	5	C,B,P
35	N	2	P,C
40	Y	9	P
45	G	5	M,L
50	S	9	L,M,N
55	R	2	N
60	T	5	Y,N
65	K	9	G,S
70	Q1	5	K,T,R

Problem 4.3 Using matrix to calculate TF and FF

NO	ACT	DUR	PREDECESSORS
5	A	10	
10	B	9	A
15	C	7	A
20	Q	9	A
25	L	1	B
30	F	5	C
35	X	4	Q,C
40	N	7	Q
45	Y	6	L
50	T	9	F,L
55	J	3	X,F,N
60	S	6	N,X
65	V	10	Y,T,J
70	U	8	V,S

Solution 4.1

SUC	A	B	C	Q	M	L	X	N	Z	S	J	T	V	Q1	EFD
DUR	4	5	3	8	1	8	2	7	8	8	6	4	10	6	
ESD	0	4	4	4	9	12	12	12	20	20	20	19	28	38	
FF	0	0	0	0											A 4
TF	0	---	---	---											
FF	0	3	5	0											B 9
TF	3				0	3									
FF	2				---	---									C 7
TF	5				10	3									
FF	0				2	5	5								Q 12
TF	0				---	---	---								
FF	10				12	5	13								M 10
TF	0				0	0	0								
FF	10				---	---	---								L 20
TF	0				0	8	15								
FF	6				10	10									X 14
TF	8				---	---	---								
FF	0				0	0	0								N 19
TF	15				---	---	---								
FF	0				0	0	2								Z 28
TF	0				6										
FF	0				8										S 28
TF	0				0										
FF	2				0										J 26
TF	2				---	---	---								
FF	15				0										T 23
TF	15				---	---	---								
FF	0				0										V 38
TF	0				---	---	---								
FF	0				0										Q1 44
TF	0				---	---	---								

Solution 4.2

	A	B	C	P	M	L	N	Y	G	S	R	T	K	Q1	
DUR	7	5	7	5	1	5	2	9	5	9	2	5	9	5	
ESD	0	7	7	7	14	14	14	12	19	19	16	21	28	37	EFD
FF	0	0	0	0											A 7
TF	0	---	---	---											
FF	2	2	0	2	2	2									B 12
TF	2				---	---									
FF	0				6	2									C 14
TF	0				0	0	0								
FF	0				---	---	---								
TF	0				4	0	3								P 12
FF	0				2	2	0								
TF	2				---	---	---								
FF	4				2	5	11								M 15
TF	4							4	4						
FF	0							---	---						
TF	0							8	4						L 19
FF	0							0	0						
TF	0							---	---						
FF	0							4	0						N 16
TF	0								3	0	5				
FF	0								---	---	---				
TF	3								3	19	16				Y 21
FF	0										0				
TF	11										---				
FF	4										11				G 24
TF	4											4			
FF	0											---			
TF	0											4			S 28
FF	19											0			
TF	19											---			
FF	11											19			R 18
TF	11												11		
FF	0												---		
TF	0												11		T 26
FF	0												0	0	K 37
TF	0												---		
FF	0												0	0	Q1 42
TF	0														

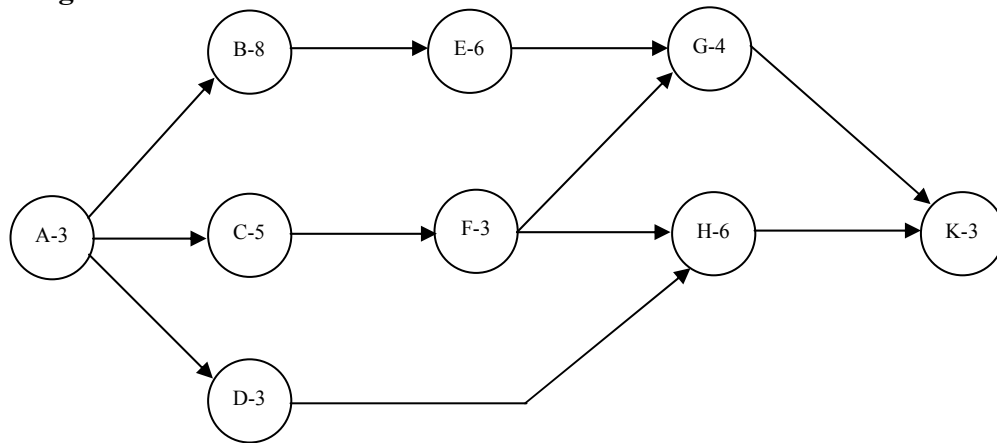
Solution 4.3

	A	B	C	Q	L	F	X	N	Y	T	J	S	V	U	EFD
DUR	10	9	7	9	1	5	4	7	6	9	3	6	10	8	
ESD		10	10	10	19	17	19	19	20	22	26	26	31	41	
FF 0	0	0	0	0											A 10
TF 0		---	---	---											
FF 0		2	0	2											B 19
TF 2					0										
FF 0					---										C 17
TF 0					2	0	2								
FF 0					---	---	---								
TF 0					0	7	7								Q 19
FF 0					0	0	0	0							
TF 2					---	---	---	---							
FF 0					5	2	5	2							L 20
TF 2					0				0	2					
FF 0					---	---	---	---							
TF 0					5	2	5	2							F 22
FF 0					0				0	4					
TF 0					---	---	---	---							
FF 3					0	6	0	6							X 23
TF 5					3				3	3					
FF 0					---	---	---	---							
TF 5					5	12	5	12							N 26
FF 0					0				0	0					
TF 2					---	---	---	---							
FF 5					2	9	2	9							Y 26
TF 5					0				5	5					
FF 0					---	---	---	---							
TF 0					0				0	0					T 31
FF 2					---	---	---	---							
TF 2					0				2	2					J 29
FF 9					2				---	---					
TF 9					2				2	2					S 32
FF 0					9				9	9					
TF 0					---	---	---	---							
FF 0					9				0	0					V 41
TF 0					---	---	---	---							
FF 0					0				0	0					U 49
TF 0					---	---	---	---							

5. PROJECT UPDATING

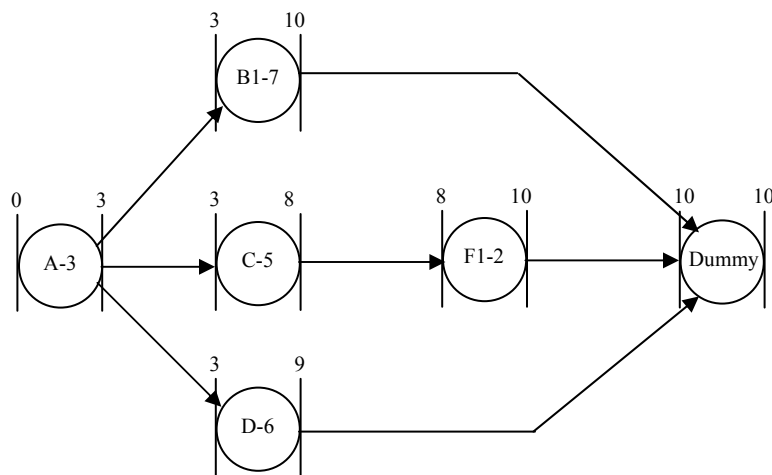
Project Updating

Original Target Schedule

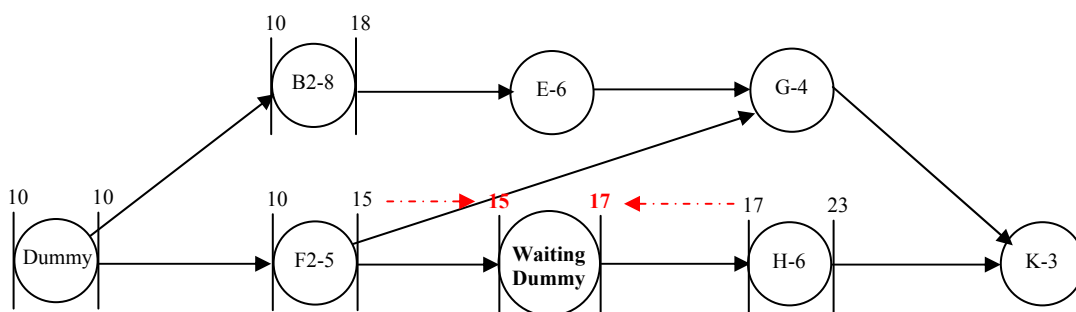


Update Information at data date = 10.

1. Activity D takes 6 days to finish.
2. Activity F takes 5 more days to complete.
3. It is expected that it will take total 15 days to finish activity B.
4. Activity H cannot start until date 17 because of the delay of material.



Please study these two networks and pay attention to on-processing activities (B and F).



6. PERT—PNET

6. Probabilistic Scheduling using PERT and PNET

Problem 6.1 Probabilistic scheduling using PERT and PNET methods

- 1.) Set up a table and calculate Early Event Times (TE), Late Event Times (TL), Activity Free Slack (AFS), and Activity Total Slack (ATS).
- 2.) Determine the PERT "critical path(s)" and the mean and standard deviation for the project duration.
- 3.) Use PNET method to find the project durations and their corresponding probabilities of project completion from 0 to 100%.

i	j	Act	a	M	b
5	10	A	5.00	7.00	9.50
5	15	B	4.50	5.00	7.00
5	20	C	6.00	8.00	10.50
5	25	D	4.00	6.00	7.50
10	30	E	2.50	3.50	5.00
10	35	F	4.00	6.00	8.00
15	35	G	6.50	8.00	11.00
15	45	H	4.00	5.00	7.00
20	40	I	2.50	3.50	4.50
20	45	J	4.00	6.00	7.00
25	45	K	2.50	4.00	5.50
30	50	L	1.00	2.00	3.00
35	50	M	3.50	4.50	6.50
40	50	N	4.00	6.00	8.00
45	55	O	4.50	5.00	7.00
50	55	Z	4.00	6.00	9.00

Problem 6.2 Probabilistic scheduling using PERT and PNET methods

- 1.) Set up a table and calculate Early Event Times (TE), Late Event Times (TL), Activity Free Slack (AFS), and Activity Total Slack (ATS).
- 2.) Determine the PERT "critical path(s)" and the mean and standard deviation for the project duration.
- 3.) Use PNET method to find the project durations and their corresponding probabilities of project completion from 0 to 100%.

i	j	Act	a	M	b
5	10	A	2.00	4.00	6.50
5	15	B	6.00	7.00	7.50
5	30	C	1.00	2.00	5.50
10	20	Y	2.50	4.50	5.50
10	25	U	5.00	6.00	7.00
15	30	I	3.00	4.00	7.50
20	35	O	2.50	5.50	8.00
25	35	P	2.00	3.00	5.00
25	40	L	3.00	4.00	4.00
30	45	K	1.00	2.00	3.50
35	50	J	5.00	6.50	8.00
40	50	H	4.00	5.00	6.50
45	55	G	2.00	3.00	5.00
50	60	F	6.50	7.50	9.50
55	60	D	3.50	5.00	7.00
60	65	S	3.00	4.00	4.50

Problem 6.3 Probabilistic scheduling using PERT and PNET methods

- 1.) Set up a table and calculate Early Event Times (TE), Late Event Times (TL), Activity Free Slack (AFS), and Activity Total Slack (ATS).
- 2.) Determine the PERT "critical path(s)" and the mean and standard deviation for the project duration.
- 3.) Use PNET method to find the project durations and their corresponding probabilities of project completion from 0 to 100%.

i	j	Act	a	M	b
5	10	S	3.00	5.00	9.00
5	15	U	5.00	6.00	8.00
10	20	P	5.00	6.00	7.00
10	25	E	3.00	4.00	7.00
15	25	R	0.00	0.00	0.00
15	30	W	4.00	7.00	9.00
20	35	O	2.00	3.00	4.00
25	35	M	4.00	6.00	11.00
30	35	A	0.00	0.00	0.00
30	40	N	5.00	6.00	9.00
35	45	X	1.00	3.00	5.00
40	45	Y	3.00	6.00	7.00

Solution 6.1

Step 1. Calculate $E[ti]$, $SD[ti]$, and $Var[ti]$.

Table 1. Activity Properties

i	j	Act	a	M	b	$E[ti]$	$SD[ti]$	$Var[ti]$	TEi	TEi+E[ti]	TLi-E[ti]	TLj	ATS	AFS
5	10	A	5.00	7.00	9.50	7.08	0.75	0.56	0	7.08	0.42	7.5	0.42	0
5	15	B	4.50	5.00	7.00	5.25	0.42	0.18	0	5.25	0	5.25	0	0
5	20	C	6.00	8.00	10.50	8.08	0.75	0.56	0	8.08	0.42	8.5	0.42	0
5	25	D	4.00	6.00	7.50	5.92	0.58	0.34	0	5.92	8.83	14.75	8.83	0
10	30	E	2.50	3.50	5.00	3.58	0.42	0.18	7.08	10.66	12.25	15.83	5.17	0
10	35	F	4.00	6.00	8.00	6	0.67	0.45	7.08	13.08	7.5	13.5	0.42	0.42
15	35	G	6.50	8.00	11.00	8.25	0.75	0.56	5.25	13.5	5.25	13.5	0	0
15	45	H	4.00	5.00	7.00	5.17	0.5	0.25	5.25	10.42	13.58	18.75	8.33	3.49
20	40	I	2.50	3.50	4.50	3.33	0.33	0.11	8.08	11.41	8.5	11.83	0.42	0
20	45	J	4.00	6.00	7.00	5.83	0.5	0.25	8.08	13.91	12.92	18.75	4.84	0
25	45	K	2.50	4.00	5.50	4	0.5	0.25	5.92	9.92	14.75	18.75	8.83	3.99
30	50	L	1.00	2.00	3.00	2	0.33	0.11	10.66	12.66	15.83	17.83	5.17	5.17
35	50	M	3.50	4.50	6.50	4.33	0.33	0.11	13.5	17.83	13.5	17.83	0	0
40	50	N	4.00	6.00	8.00	6	0.67	0.45	11.41	17.41	11.83	17.83	0.42	0.42
45	55	O	4.50	5.00	7.00	5.25	0.42	0.18	13.91	19.16	18.75	24	4.84	4.84
50	55	Z	4.00	6.00	9.00	6.17	0.83	0.69	17.83	24	17.83	24	0	0

$$E[ti] = (a+4M+b)/6$$

$$SD[ti] = (b-a)/6 \Rightarrow Var[ti] = SD[ti]^2$$

Step 2. Determine all possible paths and calculate their E[T], Var[T], and SD[T]

Table 2. Path Properties

i	path	E[Ti]	Var[Ti]	SD[Ti]	E[Ti]	
					-3SD[Ti]	+3SD[Ti]
1	5 - 15 - 35 - 50 - 55	24.00	1.54	1.24	20.28	27.72
2	5 - 20 - 40 - 50 - 55	23.58	1.81	1.35	19.53	27.63
3	5 - 10 - 35 - 50 - 55	23.58	1.81	1.35	19.53	27.63
4	5 - 20 - 45 - 55	19.16	0.99	0.99	16.19	22.13
5	5 - 10 - 30 - 50 - 55	18.83	1.54	1.24	15.11	22.55
6	5 - 15 - 45 - 55	15.67	0.61	0.78	13.33	18.01
7	5 - 25 - 45 - 55	15.17	0.77	0.88	12.53	17.81
		24.00			20.28	27.72

Example: Path 1 consists of activities 5-15, 15-35, 35-50, and 50-55.

Activity =>	5-15	15-35	35-50	50-55	
E[ti]	5.25	8.25	4.33	6.17	E[T₁] = 24
Var[ti]	0.18	0.56	0.11	0.69	Var[T₁] = 1.54

Thus, SD[T₁] of path1 is $1.54^{0.5} = 1.24$

NOTE:

1. Do not add SD[ti] of activities to get SD[T] of a path. Path SD[T] must be derived from a square root of the summation of Var[ti] of activities in the path.
2. Path 6 and 7 can be neglected because their E[Ti]+3SD[Ti], which are 18.01 and 17.81, are relatively short compared to the maximum E[T]-3SD[T], which is 20.28.
3. Paths in Table 2 must be sorted by E[T] before constructing Table 3, otherwise you might represent a longer path by a shorter one.

Step 3. Calculate correlation between paths

Table 3. Correlations between paths

r 1 2 Pij	0.41
r 1 3 Pij	0.48
r 1 4 Pij	0
r 1 5 Pij	0.45
r 1 6 Pij	0.18
r 1 7 Pij	0
r 2 3 Pij	0.38
r 2 4 Pij	0.42
r 2 5 Pij	0.41
r 2 6 Pij	0
r 2 7 Pij	0
r 3 4 Pij	0
r 3 5 Pij	0.75
r 3 6 Pij	0
r 3 7 Pij	0
r 4 6 Pij	0.23
r 4 7 Pij	0.2
r 6 7 Pij	0.26

Correlation greater than 0.5, thus eliminate path 5

$$\rho_{ij} = \frac{\sum_{k \in (\pi_i \cap \pi_j)} \sigma_k^2}{\sigma_{Ti} \sigma_{Tj}} \quad \rho_{ij} \text{ is the correlation between path i and j.}$$

$k \in (\pi_i \cap \pi_j)$ are activities that are in both path i and j.

σ_{Ti} is standard deviation of path i, SD[Ti]

σ_{Tj} is standard deviation of path j, SD[Tj]

Example: Path 1 and 2 have activities 50-55 in common (its SD[ti] is 0.83), and their SD[T] are 1.24 and 1.35. Thus,

$$\rho_{ij} = \frac{\sum_{k \in (\pi_i \cap \pi_j)} \sigma_k^2}{\sigma_{Ti} \sigma_{Tj}} = \rho_{12} = \frac{0.83^2}{1.24 \times 1.35} = 0.412$$

NOTE:

1. Paths in Table 2 must be sorted by E[T] before constructing Table 3, otherwise you might represent a longer path by shorter one.
2. Path 5 is represented by path 3. (A shorter path with high correlation to a longer path is represented by the longer one, NOT the other way around)

Step 4. Calculate probabilities of project completion

Table 4. Probability of Project Completion

T	1 (PERT)	2	3	4	6	7	all combine (PNET)
19	0.000	0.000	0.000	0.436	1.000	1.000	0.000
20	0.001	0.004	0.004	0.802	1.000	1.000	0.000
21	0.008	0.028	0.028	0.968	1.000	1.000	0.000
22	0.053	0.121	0.121	0.998	1.000	1.000	0.001
23	0.210	0.334	0.334	1.000	1.000	1.000	0.023
24	0.500	0.622	0.622	1.000	1.000	1.000	0.194
25	0.790	0.854	0.854	1.000	1.000	1.000	0.576
26	0.947	0.963	0.963	1.000	1.000	1.000	0.879
27	0.992	0.994	0.994	1.000	1.000	1.000	0.981
28	0.999	0.999	0.999	1.000	1.000	1.000	0.998
29	1.000	1.000	1.000	1.000	1.000	1.000	1.000

$$F_i(x) = Fu\left(\frac{x - E[T_i]}{\sigma_i}\right)$$

Example:

Probability of finishing the project less **than 26 days** according to path 1 is $F_1(26) = Fu\left(\frac{26 - 24}{1.24}\right) = Fu(1.61) = 0.946$

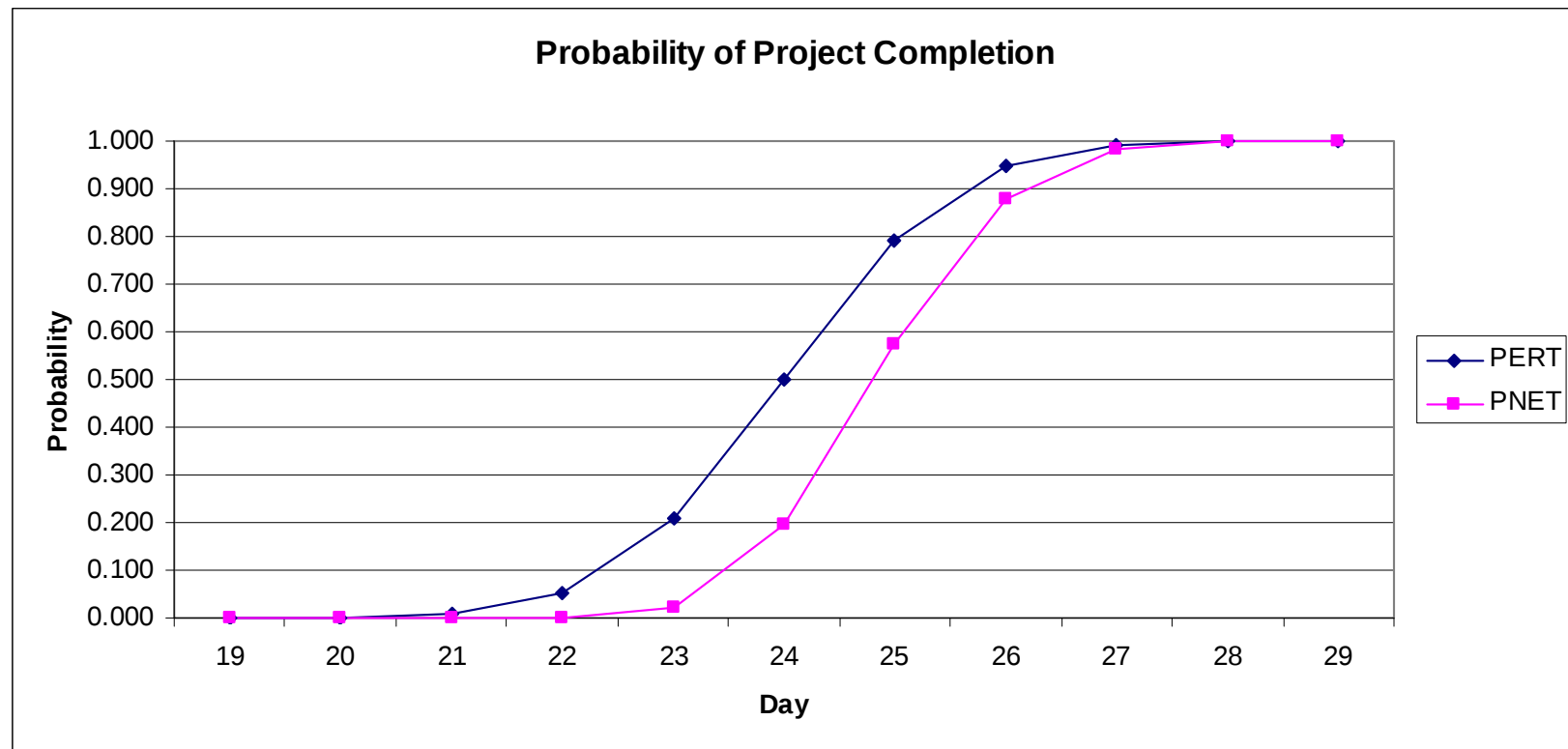
Probability of finishing the project less than 19 days according to path 1 is

$$F_1(19) = Fu\left(\frac{19 - 24}{1.24}\right) = Fu(-4) = 1 - Fu(4) = 1 - 0.99997 \approx 0$$

Probability of finishing the project **greater than 26 days** is $1 - F_1(26) = 1 - 0.946 = 0.54$

NOTE:

As shown in Table 4, path 6 and 7 can be ignored since their $E[T_i] + 3SD[T_i]$ (from Table 2), which are 18.01 and 17.81 respectively, are less than the maximum $E[T_i] - 3SD[T_i]$, 20.28. In Table 4, agreeing to Table 2, probabilities of path 6 and 7 are all equal to 1.



Solution 6.2

Step 1. Calculate $E[t_i]$, $SD[t_i]$, and $Var[t_i]$.

Table 1. Activity Properties

i	j	Act	a	M	b	$E[t_i]$	$SD[t_i]$	$Var[t_i]$	TE_i	$TE_i + E[t_i]$	$TL_i - E[t_i]$	TL_j	ATS	AFS
5	10	A	2.00	4.00	6.50	4.08	0.75	0.56	0	4.08	0	4.08	0	0
5	15	B	6.00	7.00	7.50	6.92	0.25	0.06	0	6.92	6.74	13.66	6.74	0
5	30	C	1.00	2.00	5.50	2.33	0.67	0.45	0	2.33	15.5	17.83	15.5	8.76
10	20	Y	2.50	4.50	5.50	4.25	0.58	0.34	4.08	8.33	4.08	8.33	0	0
10	25	U	5.00	6.00	7.00	6	0.33	0.11	4.08	10.08	4.58	10.58	0.5	0
15	30	I	3.00	4.00	7.50	4.17	0.5	0.25	6.92	11.09	13.66	17.83	6.74	0
20	35	O	2.50	5.50	8.00	5.42	0.92	0.85	8.33	13.75	8.33	13.75	0	0
25	35	P	2.00	3.00	5.00	3.17	0.5	0.25	10.08	13.25	10.58	13.75	0.5	0.5
25	40	L	3.00	4.00	4.00	3.83	0.17	0.03	10.08	13.91	11.67	15.5	1.59	0
30	45	K	1.00	2.00	3.50	2.08	0.42	0.18	11.09	13.17	17.83	19.91	6.74	0
35	50	J	5.00	6.50	8.00	6.83	0.83	0.69	13.75	20.58	13.75	20.58	0	0
40	50	H	4.00	5.00	6.50	5.08	0.42	0.18	13.91	18.99	15.5	20.58	1.59	1.59
45	55	G	2.00	3.00	5.00	3.17	0.5	0.25	13.17	16.34	19.91	23.08	6.74	0
50	60	F	6.50	7.50	9.50	7.58	0.58	0.34	20.58	28.16	20.58	28.16	0	0
55	60	D	3.50	5.00	7.00	5.08	0.58	0.34	16.34	21.42	23.08	28.16	6.74	6.74
60	65	S	3.00	4.00	4.50	3.92	0.25	0.06	28.16	32.08	28.16	32.08	0	0

Step 2. Determine all possible paths and calculate their $E[T]$, $Var[T]$, and $SD[T]$

Table 2. Path Properties

i	path	$E[T_i]$	$Var[T_i]$	$SD[T_i]$	$E[T_i]$	
					$-3SD[T_i]$	$+3SD[T_i]$
1	5 - 10 - 20 - 35 - 50 - 60 - 65	32.08	2.84	1.69	27.01	37.15
2	5 - 10 - 25 - 35 - 50 - 60 - 65	31.58	2.01	1.42	27.32	35.84
3	5 - 10 - 25 - 40 - 50 - 60 - 65	30.49	1.28	1.13	27.1	33.88
4	5 - 15 - 30 - 45 - 55 - 60 - 65	25.34	1.14	1.07	22.13	28.55
5	5 - 30 - 45 - 55 - 60 - 65	16.58	1.28	1.13	13.19	19.97
		32.08			27.32	37.15

Step 3. Calculate correlation between paths

Table 3. Correlations between paths

r 1 2 Pij	0.69
r 1 3 Pij	0.5
r 1 4 Pij	0.03
r 1 5 Pij	0.03
r 3 4 Pij	0.05
r 3 5 Pij	0.05
r 4 5 Pij	0.68

Step 4. Calculate probabilities of project completion

Table 4. Probability of Project Completion

T	1 (PERT)	3	4	all combine (PNET)
26	0.000	0.000	0.731	0.000
27	0.001	0.001	0.940	0.000
28	0.008	0.014	0.994	0.000
29	0.034	0.094	1.000	0.003
30	0.109	0.332	1.000	0.036
31	0.261	0.674	1.000	0.176
32	0.481	0.909	1.000	0.437
33	0.707	0.987	1.000	0.698
34	0.872	0.999	1.000	0.871
35	0.958	1.000	1.000	0.958
36	0.990	1.000	1.000	0.990
37	0.998	1.000	1.000	0.998
38	1.000	1.000	1.000	1.000

Problem 6.3

Table 1.

i	j	Act	a	M	b	E[ti]	SD[ti]	Var[ti]	TEi	TEi+E[ti]	TLi-E[ti]	TLj	ATS	AFS
5	10	S	3.00	5.00	9.00	5.33	1	1	0	5.33	5.84	11.17	5.84	0
5	15	U	5.00	6.00	8.00	6.17	0.5	0.25	0	6.17	0	6.17	0	0
10	20	P	5.00	6.00	7.00	6	0.33	0.11	5.33	11.33	13	19	7.67	0
10	25	E	3.00	4.00	7.00	4.33	0.67	0.45	5.33	9.66	11.17	15.5	5.84	0
15	25	R	0.00	0.00	0.00	0	0	0	6.17	6.17	15.5	15.5	9.33	3.49
15	30	W	4.00	7.00	9.00	6.83	0.83	0.69	6.17	13	6.17	13	0	0
20	35	O	2.00	3.00	4.00	3	0.33	0.11	11.33	14.33	19	22	7.67	1.83
25	35	M	4.00	6.00	11.00	6.5	1.17	1.37	9.66	16.16	15.5	22	5.84	0
30	35	A	0.00	0.00	0.00	0	0	0	13	13	22	22	9	3.16
30	40	N	5.00	6.00	9.00	6.33	0.67	0.45	13	19.33	13	19.33	0	0
35	45	X	1.00	3.00	5.00	3	0.67	0.45	16.16	19.16	22	25	5.84	5.84
40	45	Y	3.00	6.00	7.00	5.67	0.67	0.45	19.33	25	19.33	25	0	0

Table 2.

i	PATH	E[Ti]	Var[Ti]	SD[Ti]	E[Ti]	
					-3SD[Ti]	+3SD[Ti]
1	5 - 15 - 30 - 40 - 45	25.00	1.84	1.36	20.92	29.08
2	5 - 10 - 25 - 35 - 45	19.16	3.27	1.81	13.73	24.59
3	5 - 10 - 20 - 35 - 45	17.33	1.67	1.29	13.46	21.20
4*	5 - 15 - 30 - 35 - 45	16.00	1.39	1.18	12.46	19.54
5*	5 - 15 - 25 - 35 - 45	15.67	2.07	1.44	11.35	19.99
		25.00			20.92	29.08

NOTE:

1. Data in Table 2 must be sorted by E[T] before constructing Table 3, otherwise you might try to represent a longer path by shorter one.
2. Path 4 and 5 can be ignored since their $E[Ti]+3SD[Ti]$, which are 19.54 and 19.99 respectively, are less than the maximum $E[Ti]-3SD[Ti]$, 20.92.

Table 3.

r 1 2 Pij	0.00
r 1 3 Pij	0.00
r 1 4 Pij	0.59
r 1 5 Pij	0.13
r 2 3 Pij	0.62
r 2 5 Pij	0.70

Table 4.

T	1 (PERT)	2	all combine (PNET)
20	0.000	0.679	0.000
21	0.002	0.845	0.001
22	0.014	0.942	0.013
23	0.071	0.983	0.070
24	0.231	0.996	0.230
25	0.500	0.999	0.500
26	0.769	1.000	0.769
27	0.929	1.000	0.929
28	0.986	1.000	0.986
29	0.998	1.000	0.998
30	1.000	1.000	1.000

NOTE:

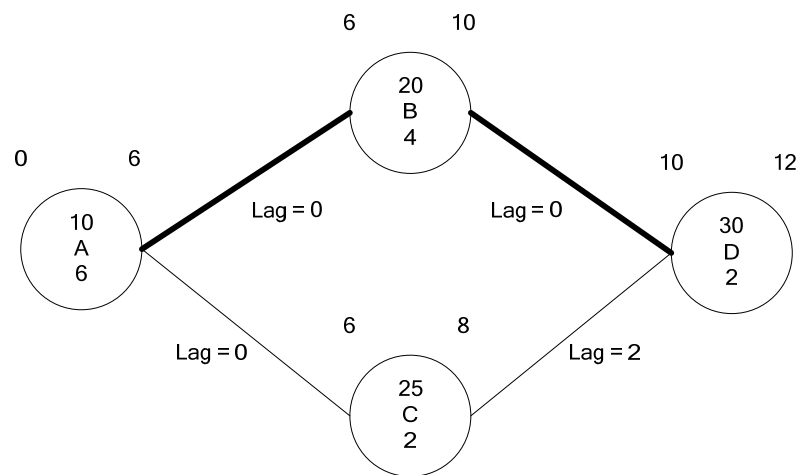
Path 4 can be represented by path 1, and also path 3 can be represented by path 2. (A shorter path with high correlation to a longer path is represented by the longer one, NOT the other way around)

7. TIME-COST TRADEOFF TABLES

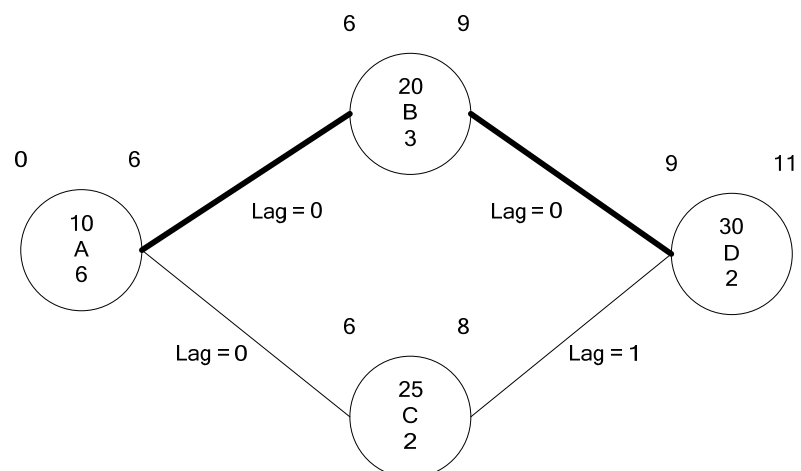
Time-Cost Tradeoff

The following is a basic idea about Time-Cost Tradeoff. Read it carefully and think along the way will give you an insight idea about this subject.

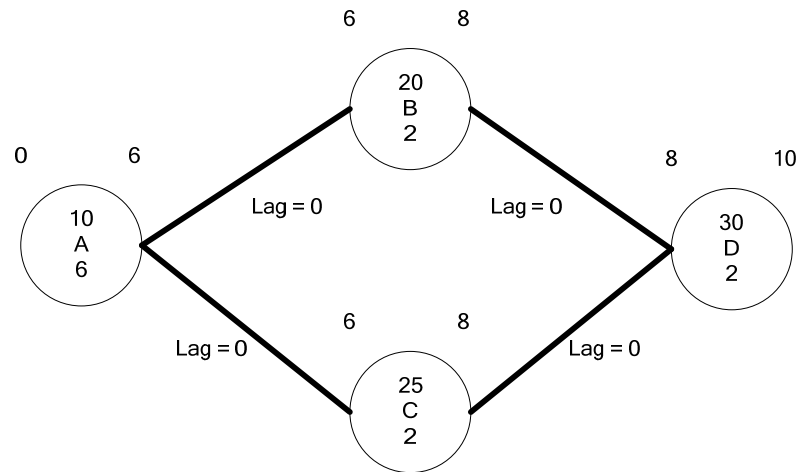
If the minimum duration of B is 1 day (TC=1), and cost of shortening its duration is \$200 per 1 shortened day, how many days will you shorten activity B?



1) Shortening activity B by 1 day costs \$200 and reduces project duration to 11 days.



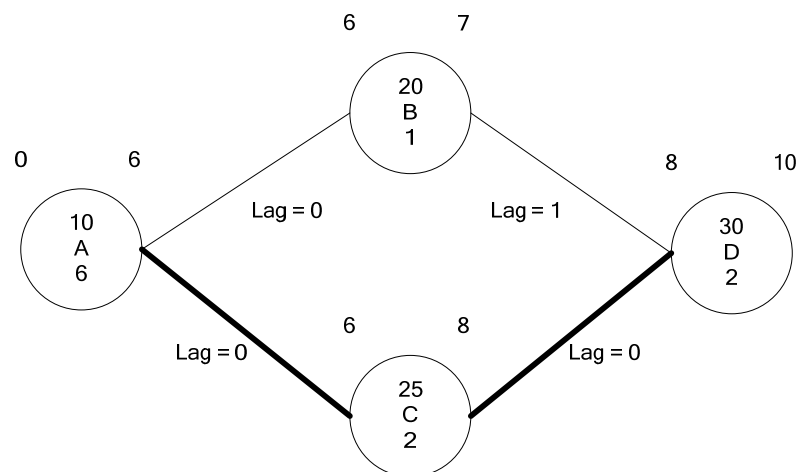
2) Shortening activity B by 2 days costs \$400 and reduces project duration to 10 days.



As shown in the case of shortening B by 2 days, one of the lags, between C and D, becomes 0, because shortening B allows D to start earlier. As a result, the network now has 2 critical paths. Accordingly, shortening activity B alone from this point does not reduce project duration because another critical path A-C-D still control the project duration (10 days). So, if you want to shorten B beyond this point, you have to shorten activity C as well.

It should be noted that when one of the lags becomes 0 due to shortening an activity, it is not necessary that the new 0 lag link will incur a new critical path. However, it is a good practice to stop shortening an activity at the point when one of the lags becomes 0, because we do not know whether there is a new critical path or not.

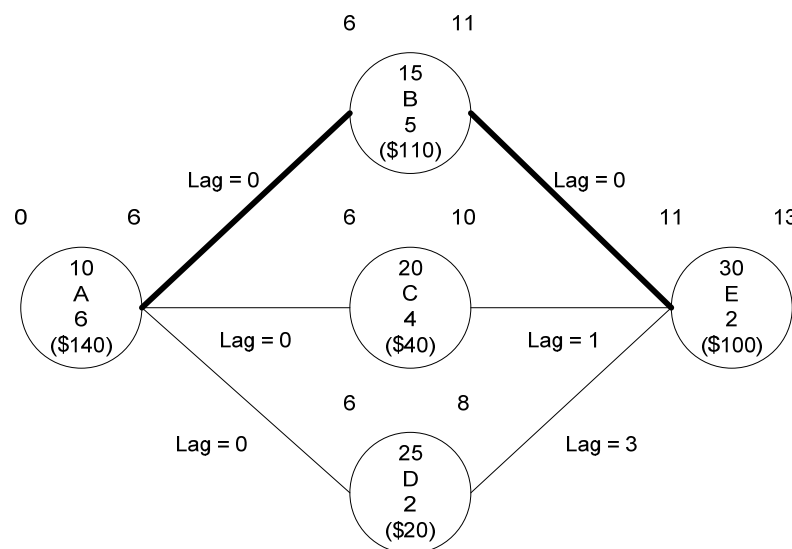
3) Shorten activity B by 3 days costs \$600 and project duration is still the same as shortening B by 2 days.



What happens here is after shorten B by 2 days, path A-B-D and path A-C-D become critical paths. Shortening B further (the 3rd day) cannot reduce project duration because ANOTHER CRITICAL PATH (A-C-D) still dominates the project duration. This means the \$200 that is spent on the 3rd shortened day is wasted, since we do not gain any reduction in project duration from the last \$200.

2) What will you do if you have the following options? And in which orders?

- Activity A can be crashed to 3 days by paying extra \$140 per 1 shorten day.
- Activity B can be crashed to 1 day by paying extra \$110 per 1 shorten day.
- Activity C can be crashed to 3 days by paying extra \$40 per 1 shorten day.
- Activity D can be crashed to 1 day by paying extra \$20 per 1 shorten day.
- Activity E can be crashed to 1 day by paying extra \$100 per 1 shorten day.



Procedure

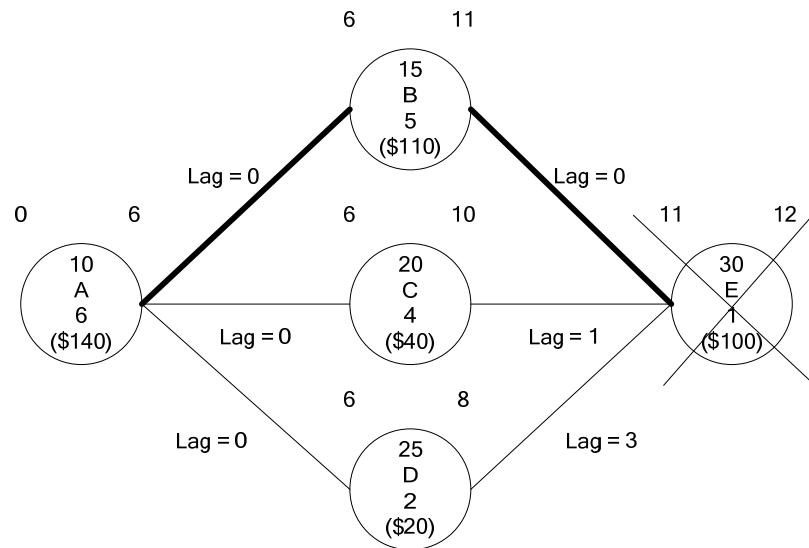
First, select a critical activity with the cheapest \$/shortened day. (If you don't know why we select a critical activity, I suggest you to read the BABY problem again. Please make sure you are sober this time.)

Second, shorten the selected activity until one of the lags in the network becomes 0.

Thus,

Step 1 (or Cycle 1)

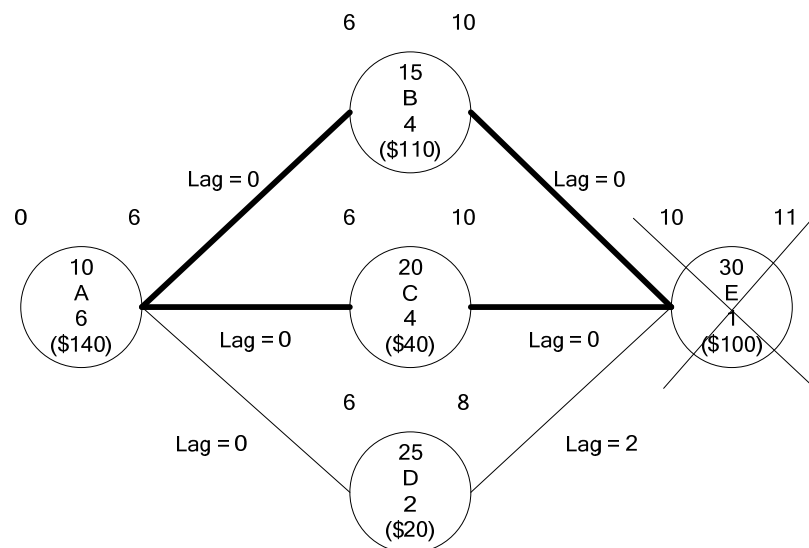
Shorten E by 1 day. There is no change in lags. Project duration becomes 12 days.



Step 2 (or Cycle 2)

Select activity B because it is a critical activity with the cheapest shortening cost (\$110).

At this step, we will shorten B by 1 day only because lag C-E will become 0.



As shown in the figure above, after shorten B, the network now has 2 critical paths, A-B-E and A-C-E.

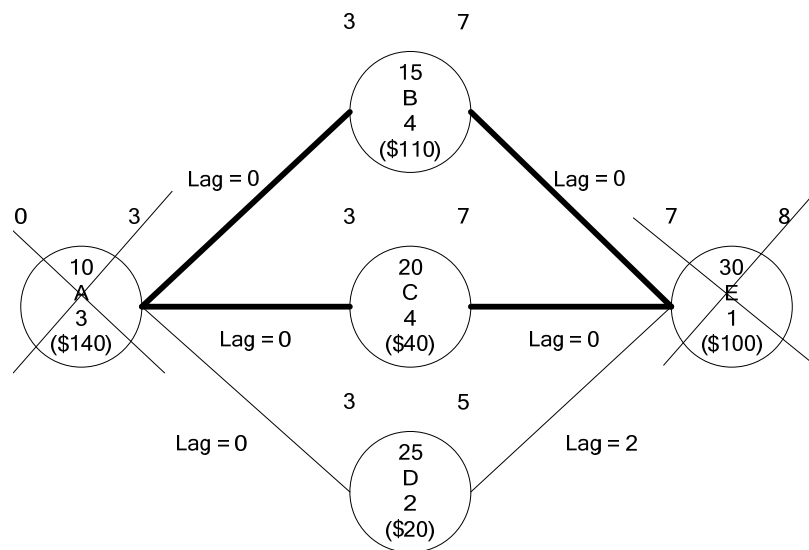
Step 3 (or Cycle 3)

At this step, we have 2 choices:

- 1) Shorten activity A alone for \$140/shortened day.
- 2) Shorten activities B and C together for $\$110 + \$40 = \$150$ /shortened day.

If you shorten either B or C alone, you are going to waste your money because project duration will remain the same.

Thus, we choose the cheaper one between the two choices, which is shortening activity A by 3 days.



Step 4 (or cycle 4)

We have to shorten B and C at the same time. WHY???

Otherwise, project duration will still be the same, which means you spend more money on the project without gaining any reduction in project duration.

Network limit is 2 days. WHY???

Lag D-E will become 0.

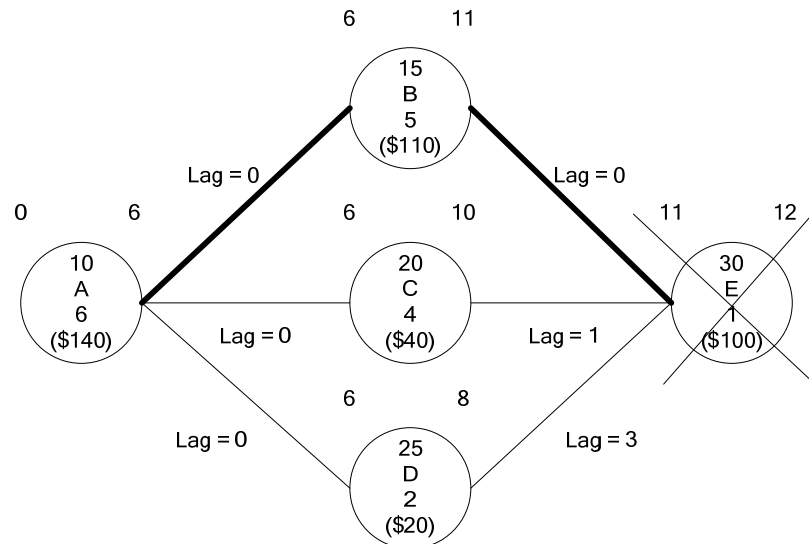
Although network limit is 2 days, we can only shorten B and C by 1 day in this cycle. WHY???

TC of activity C is 3 days.

At this point, you have an idea about time-cost tradeoff. In the next section, we will talk about how to do this in a more systematic manner by using table.

Example from the previous problem by using table to check network limit and update lags.
IMPORTANT: Column i refers to the finish date of activity, whereas column j refers to the start date of activity!!! (this is really important.)

I skip cycle 1 because it is easy. In cycle2, we decide to shorten activity B because it is the cheapest critical activity.



CYCLE			2.1			2.2			2.3		
ACTS Changed			B			B			B		
Network Limit									1		
Days Changed									1		
i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag
10	15	0									
10	20	0									
10	25	0									
15	30	0	<<			<<	<		<<	<	
20	30	1								<	0
25	30	3								<	2

In cycle 2.1, we identify the being shortened activity B by putting double arrow "<<" in this cycle 2.

In cycle 2.2, since the lag 15-30 is 0, we assume that finish date of B (or column i of B) controls the start date of E (or column j of E). Thus, shortening the finish date of B (or moving the finish date forward) will move the start date of E forward too!! Accordingly, we put an arrow for column j of E.

In cycle 2.3, since there are 3 cells in column j of E, which refers to the same thing (start date of E), we need to put an arrow for each one of them.

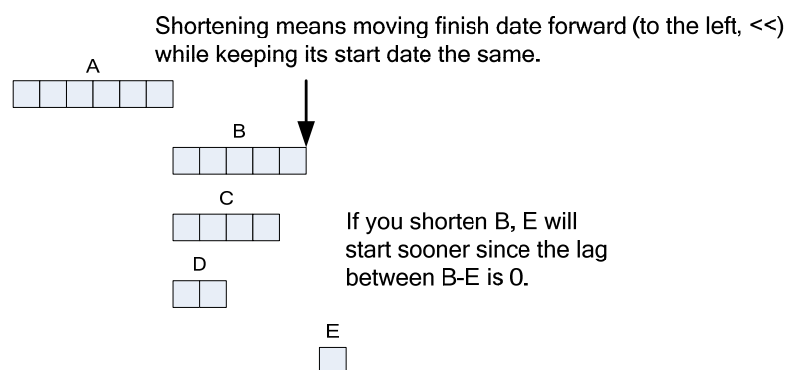
Now let's focus on one particular lag.

Part of the above table focus on lag 20-30 from cycle 2.3.

i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag
20	30	1								<	0

This unpaired arrow in cycle 2.3 on lag 20-30 means

- 1) The finish date of 20 will not be moved forward since there is no arrow in the column i of 20.
- 2) The start date of 30 (column j of 30) will be moved forward.
- 3) While keeping the finish date of 20 the same and moving the start date of 30 forward, we can only move start date of 30 forward by 1 day only because lag 20-30 will become 0. And this 1 day is the network limit for cycle 2.



Let's see if you understand the underlying idea of the table and arrow.

What do these following parts of time-cost tradeoff table mean?

Lag	i	j
3	<	<

The finish date of activity i and the start date of activity j are moved forward for the same amount. Thus, the lag between them is still the same.

Lag	i	j
2	<	

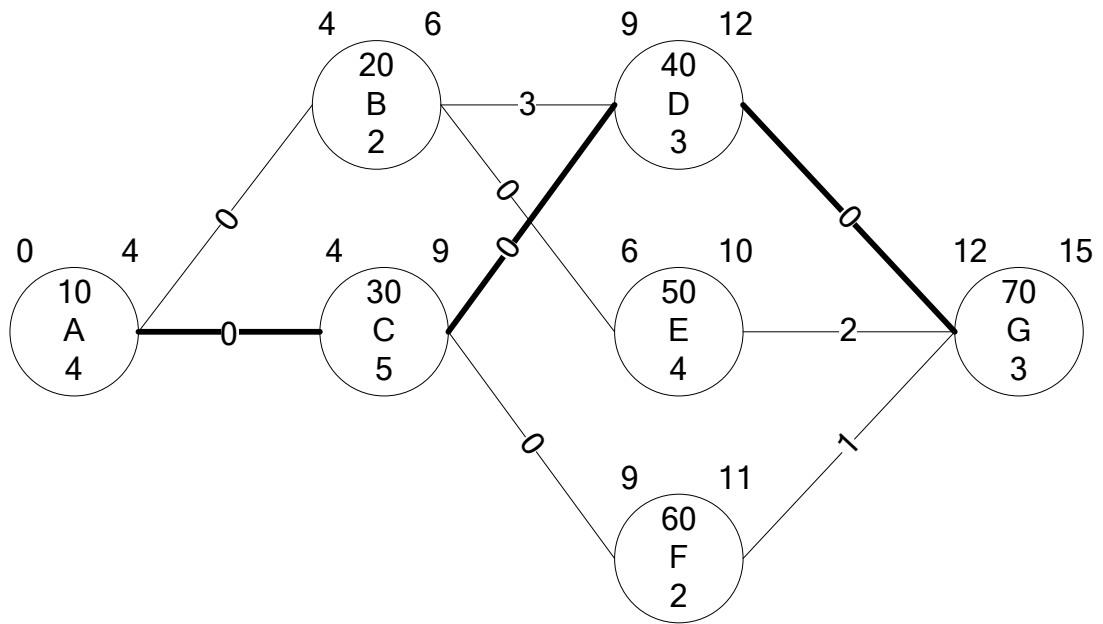
The finish date of act i is moved forward while the start date of j is still the same. Thus, the lag will increase.

Lag	i	j
0		<

The start date of activity j is moved forward while the finish date of i is still the same. Since the lag between them is 0, this is IMPOSSIBLE. You have to CROSS the arrow for activity j out, indicating that it cannot be moved. We will talk about this again later in the following example.

SETUP TABLE

	ID	TC	TN	CC	CN	T
A	10	2	4	600	300	4
B	20	1	2	800	650	2
C	30	3	5	480	240	5
D	40	1	3	300	100	3
E	50	2	4	1600	1000	4
F	60	1	2	1150	1100	2
G	70	2	3	500	300	3
			sum	5430	3690	



TALLY OF CRITICAL PATH

[illegible]

[illegible][illegible]

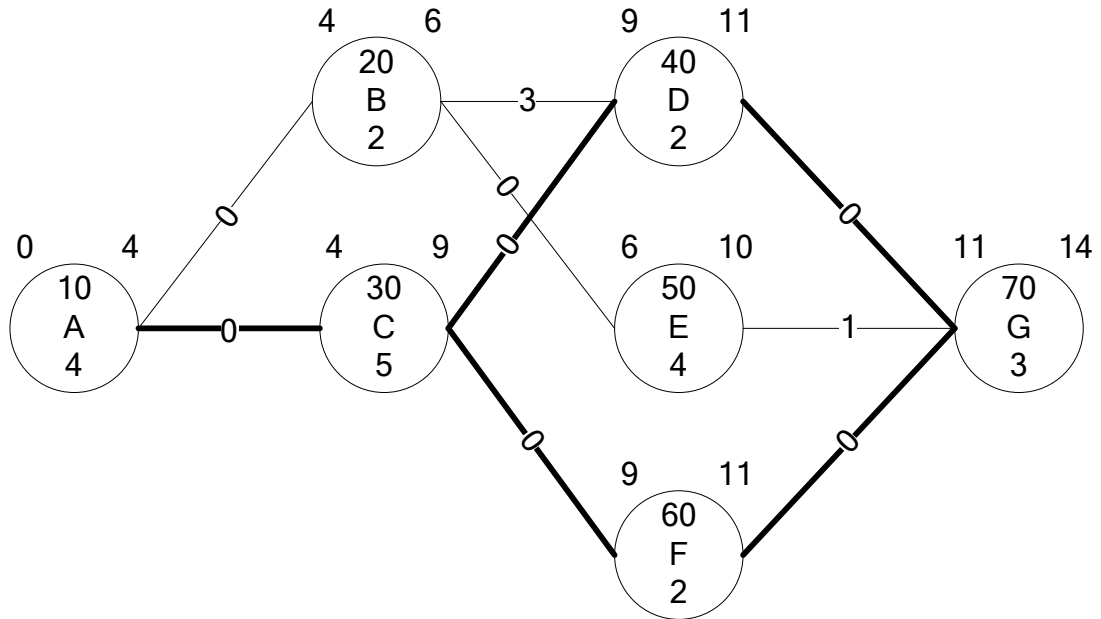
CEE536—Example Problems

[illegible]

CYCLE			1.1			1.2			1.3			1.4														
ACTS Changed			40			40			40			40														
Network Limit												1														
Days Changed												1														
i	j	Lag	i	j	Lag																					
10	20	0																								
10	30	0																								
20	40	3																								
20	50	0																								
30	40	0																								
30	60	0																								
40	70	0	<<			<<	<		<<	<		<<	<													
50	70	2								<			<	1												
60	70	1								<			<	0	This is the network limit.											

CEE536—Example Problems

	ID	TC	TN	CC	CN	T
A	10	2	4	600	300	4
B	20	1	2	800	650	2
C	30	3	5	480	240	5
D	40	1	3	300	100	2
E	50	2	4	1600	1000	4
F	60	1	2	1150	1100	2
G	70	2	3	500	300	3
			sum	5430	3690	3790
Current Project Cost = 3690 + 100 = \$3790 at 14 days with 2 critical paths						



TALLY OF CRITICAL PATH

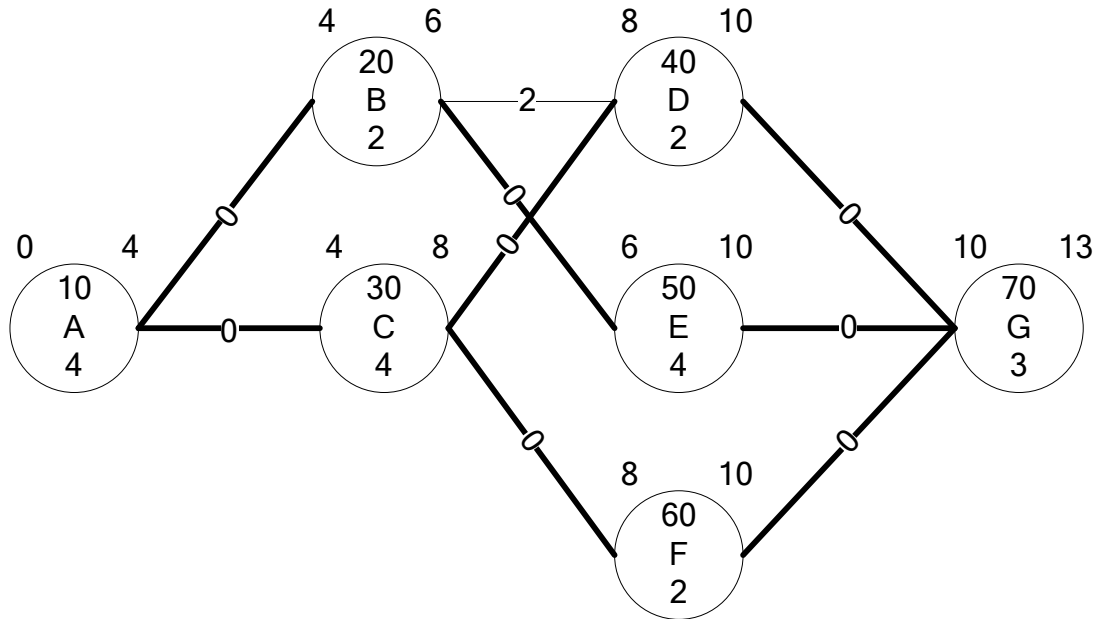
STEP	ACTIVITIES	0	1	1															
1	10	10	10	10															
2	20,30	30	30	30															
3	40,50,60	40	40	60															
4	70	70	70	70															

[illegible]

CYCLE			1			2.1			2.2			2.3			2.4			2.5			2.6			2.7		
ACTS Changed			40			30			30			30			30			30			30			30		
Network Limit			1																		1					
Days Changed			1																		1					
i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag
10	20	0																								
10	30	0																								
20	40	3											<			<			<			<			<	2
20	50	0																								
30	40	0				<<			<<	<		<<	<		<<	<		<<	<		<<	<		<<	<	
30	60	0				<<			<<			<<			<<	<		<<	<		<<	<		<<	<	
40	70	0	<<	<								<			<			<			<	<		<	<	
50	70	2		<	1																				<	0
60	70	1		<	0													<			<			<	<	

CEE536—Example Problems

	ID	TC	TN	CC	CN	T
A	10	2	4	600	300	4
B	20	1	2	800	650	2
C	30	3	5	480	240	4
D	40	1	3	300	100	2
E	50	2	4	1600	1000	4
F	60	1	2	1150	1100	2
G	70	2	3	500	300	3
			sum	5430	3690	3910
Current Project Cost = 3790 + 120 = 3910 at 14 days with 3 critical paths						



TALLY OF CRITICAL PATH

STEP	ACTIVITIES	0	1	1	2	2	2													
1	10	10	10	10	10	10	10													
2	20,30	30	30	30	30	30	20													
3	40,50,60	40	40	60	40	60	50													
4	70	70	70	70	70	70	70													

Combinations for cycle 3

1) Activity 10

10	10	10
30	30	20
40	60	50
70	70	70

2) Activities 20 and 30

10	10	10
30	30	20
40	60	50
70	70	70

3) Activities 40,50, and 60

10	10	10
30	30	20
40	60	50
70	70	70

4) Activity 70

10	10	10
30	30	20
40	60	50
70	70	70

5) Activities 30 and 50

10	10	10
30	30	20
40	60	50
70	70	70

6) Activities 20,40, and 60

10	10	10
30	30	20
40	60	50
70	70	70

7) Activities 10 and 20

10	10	10
30	30	20
40	60	50
70	70	70

8) Activities 30 and 40

10	10	10
30	30	20
40	60	50
70	70	70

9) Activities 40 and 50

10	10	10
30	30	20
40	60	50
70	70	70

Combinations 1 to 4 are considered as the obvious combinations, since you just pick critical activities from each sequence step.

Combinations 5 and 6 are a little bit harder to spot. However, if you study the network carefully, you should be able to identify them.

Combination 7 cannot be used since it will shorten 2 critical activities (10 and 20) on the same critical path (3rd path) twice.

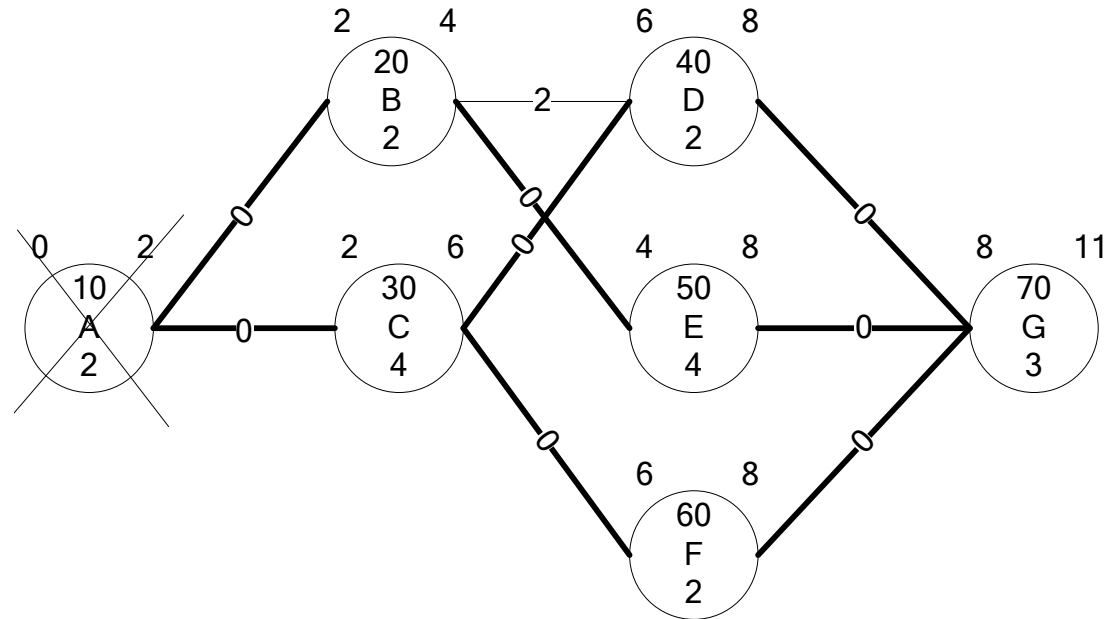
Combination 8 cannot be used since it will shorten 2 critical activities (30 and 40) on the same critical path (1st path) twice. Moreover, it does not shorten the 3rd critical path.

Combination 9 cannot be used since it does not shorten any critical activity on the 2nd critical path.

[illegible][illegible]

CEE536—Example Problems

	ID	TC	TN	CC	CN	T
A	10	2	4	600	300	2
B	20	1	2	800	650	2
C	30	3	5	480	240	4
D	40	1	3	300	100	2
E	50	2	4	1600	1000	4
F	60	1	2	1150	1100	2
G	70	2	3	500	300	3
			sum	5430	3690	4210
Project Cost = 3910 + 2 x 150 = 4210 at 14 days with 3 critical paths. Activity A cannot be crashed anymore.						



TALLY OF CRITICAL PATH

STEP	ACTIVITIES	0	1	1	2	2	2	3	3	3										
1	10	10	10	10	10	10	10	40	40	40										
2	20,30	30	30	30	30	30	20	30	30	20										
3	40,50,60	40	40	60	40	60	50	40	60	50										
4	70	70	70	70	70	70	70	70	70	70										

ACTIVITY SELECTION AND TALLY TABLE

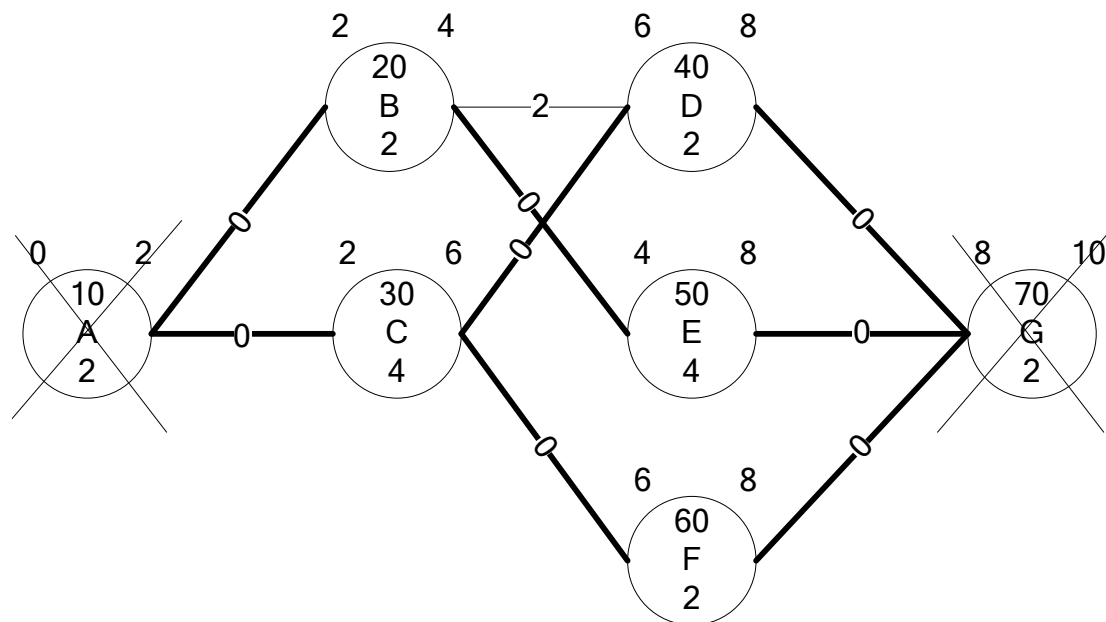
ACT	Slope	1	1	1	1	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4						
60	50					X						X			X		X			X							
40	100			X		X						X			X		X			X							
30	120		X				X				X			X		X				X							
10	150	X						X		X																	
20	150										X				X	X					X						
70	200				X				X				X						X								
50	300											X		X			X			X							
		300	240	100	200	150	120	150	200	150	270	450	200	420	300	270	450	200	420	300							

NETWORK LIMIT DETERMINATION TABLE

CYCLE			1			2			3			4																	
ACTS Changed			40			30			10			70																	
Network Limit			1			1			none			none																	
Days Changed			1			1			2			1																	
i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag															
10	20	0							<<	<																			
10	30	0							<<	<																			
20	40	3					<	2	<	<																			
20	50	0							<	<																			
30	40	0				<<	<		<	<																			
30	60	0				<<	<		<	<																			
40	70	0	<<	<		<	<		<	<																			
50	70	2		<	1		<	0	<	<																			
60	70	1		<	0	<	<		<	<																			

Cycle 5

	ID	TC	TN	CC	CN	T
A	10	2	4	600	300	2
B	20	1	2	800	650	2
C	30	3	5	480	240	4
D	40	1	3	300	100	2
E	50	2	4	1600	1000	4
F	60	1	2	1150	1100	2
G	70	2	3	500	300	2
			sum	5430	3690	4410
Project Cost = 4210 + 200 = 4410 at 14 days with 3 critical paths. Act 70 cannot be crashed anymore.						



TALLY OF CRITICAL PATH

STEP	ACTIVITIES	0	1	1	2	2	2	3	3	3	4	4	4								
1	10	10	10	10	10	10	10	10	10	10	10	10	10								
2	20,30	30	30	30	30	30	20	30	30	20	30	30	20								
3	40,50,60	40	40	60	40	60	50	40	60	50	40	60	50								
4	70	70	70	70	70	70	70	70	70	70	70	70	70								

ACTIVITY SELECTION AND TALLY TABLE

ACT	Slope	1	1	1	1	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5		
60	50					X						X			X		X			X		X		X		
40	100			X		X						X			X		X			X		X		X		
30	120		X				X				X			X		X			X		X		X			
10	150	X						X		X																
20	150										X				X	X				X	X			X		
70	200				X				X				X					X								
50	300											X		X			X		X			X	X			
		300	240	100	200	150	120	150	200	150	270	450	200	420	300	270	450	200	420	300	270	450	420	300		

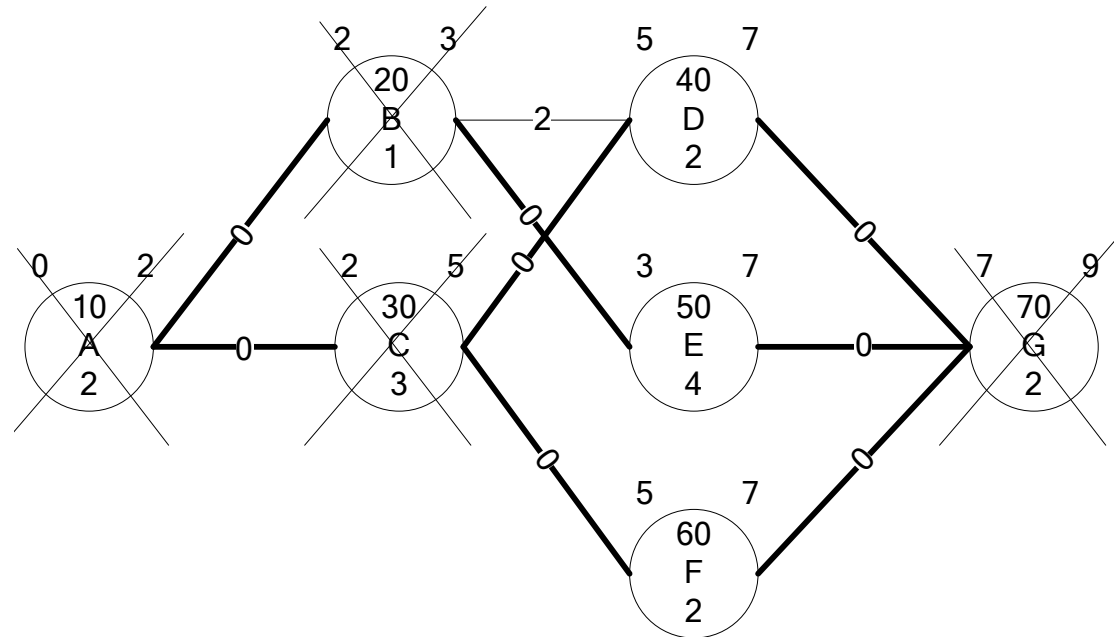
NETWORK LIMIT DETERMINATION TABLE

CYCLE			1			2			3			4			5.1			5.2			5.3			5.4			5.5		
ACTS Changed			40			30			10			70			20,30			20,30			20,30			20,30			20,30		
Network Limit			1			1			none			none															none		
Days Changed			1			1			2			1															1		
i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag
10	20	0							<<	<																			
10	30	0							<<	<																			
20	40	3					<	2	<	<					<<			<<			<<	<		<<	<		<<	<	2
20	50	0							<	<					<<			<<	<		<<	<		<<	<		<<	<	
30	40	0				<<	<		<	<					<<			<<			<<	<		<<	<		<<	<	
30	60	0				<<	<		<	<					<<			<<			<<			<<	<		<<	<	
40	70	0	<<	<		<	<		<	<											<			<			<	<	
50	70	2		<	1		<	0	<	<								<			<			<			<	<	
60	70	1		<	0	<	<		<	<														<			<	<	

Since the network limit is NONE, the days changed is the minimum possible shorten days between activity 20 and 30. However, they are the same in this case.

Cycle 6

	ID	TC	TN	CC	CN	T
A	10	2	4	600	300	2
B	20	1	2	800	650	1
C	30	3	5	480	240	3
D	40	1	3	300	100	2
E	50	2	4	1600	1000	4
F	60	1	2	1150	1100	2
G	70	2	3	500	300	2
			sum	5430	3690	4680
Project Cost = 4210 + 270 = 4680 at 14 days with 3 critical paths. Act 20 and 30 cannot be crashed anymore.						



TALLY OF CRITICAL PATH

STEP	ACTIVITIES	0	1	1	2	2	2	3	3	3	4	4	4	5	5	5						
1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10						
2	20,30	30	30	30	30	30	20	30	30	20	30	30	20	30	30	20						
3	40,50,60	40	40	60	40	60	50	40	60	50	40	60	50	40	60	50						
4	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70						

ACTIVITY SELECTION AND TALLY TABLE

ACT	Slope	1	1	1	1	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	6	
60	50					X						X			X		X			X		X		X	X	
40	100			X		X						X			X		X			X		X		X	X	
30	120		X				X				X			X		X			X		X		X			
10	150	X						X		X																
20	150										X				X	X				X	X			X		
70	200				X				X				X					X								
50	300											X		X			X		X			X	X		X	
		300	240	100	300	150	120	150	300	150	270	450	200	420	300	270	450	200	420	300	270	450	420	300	450	

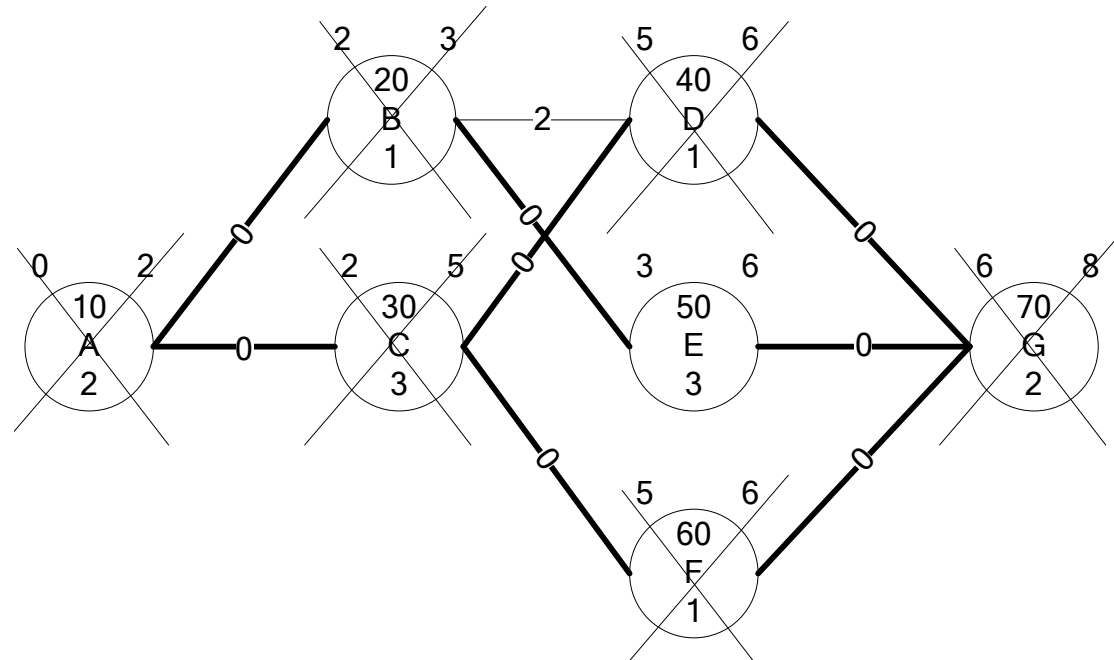
NETWORK LIMIT DETERMINATION TABLE

CYCLE			1			2			3			4			5			6								
ACTS Changed			40			30			10			70			20,30			40,50,60								
Network Limit			1			1			none			none			none			none								
Days Changed			1			1			2			1			1			1								
i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag						
10	20	0							<<	<																
10	30	0							<<	<																
20	40	3					<	2	<	<					<<	<										
20	50	0							<	<					<<	<										
30	40	0				<<	<		<	<					<<	<										
30	60	0				<<	<		<	<					<<	<										
40	70	0	<<	<		<	<		<	<					<	<		<<	<							
50	70	2		<	1		<	0	<	<					<	<		<<	<							
60	70	1		<	0	<	<		<	<					<	<		<<	<							

Since the network limit is NONE, the days changed is the minimum possible shorten days between activity 40, 50 and 60. Thus, the days changed are 1 days.

Cycle 7

	ID	TC	TN	CC	CN	T
A	10	2	4	600	300	2
B	20	1	2	800	650	1
C	30	3	5	480	240	3
D	40	1	3	300	100	1
E	50	2	4	1600	1000	3
F	60	1	2	1150	1100	1
G	70	2	3	500	300	2
			sum	5430	3690	5130
Project Cost = 4680 + 450 = 5130 at 14 days with 3 critical paths. Act 40 and 60 cannot be crashed anymore.						



TALLY OF CRITICAL PATH

STEP	ACTIVITIES	0	1	1	2	2	2	3	3	3	4	4	4	5	5	5	6	6	6			
1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10			
2	20,30	30	30	30	30	30	20	30	30	20	30	30	20	30	30	20	30	30	20			
3	40,50,60	40	40	60	40	60	50	40	60	50	40	60	50	40	60	50	40	60	50			
4	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70			

At this point it is impossible to reduce project duration since the only activity that can be shortened is activity 50.
 So we cannot satisfy the condition of selecting only one critical activity from each critical path.

ACTIVITY SELECTION AND TALLY TABLE

ACT	Slope	1	1	1	1	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	6	7
60	50					X						X			X		X			X		X		X	X	
40	100			X		X						X			X		X			X		X		X	X	
30	120		X				X				X			X		X			X		X		X			
10	150	X						X		X																
20	150										X				X	X				X	X			X		
70	200				X				X				X					X								
50	300											X		X			X		X			X	X		X	X
		300	240	100	200	150	120	150	200	150	270	450	200	420	300	270	450	200	420	300	270	450	420	300	450	300

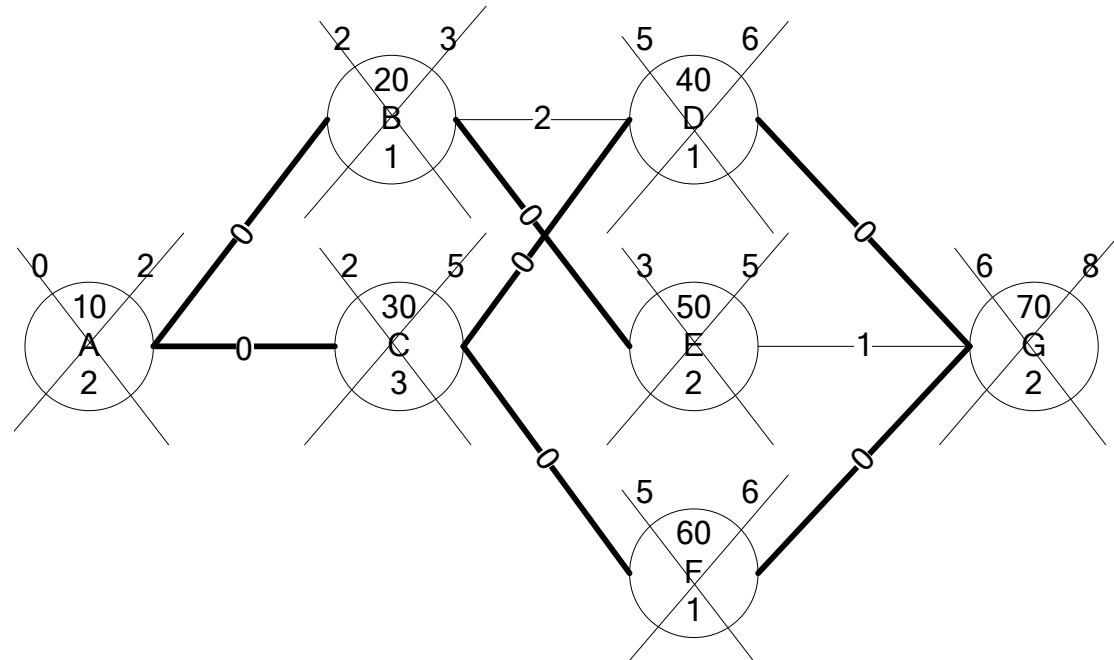
NETWORK LIMIT DETERMINATION TABLE

CYCLE			1			2			3			4			5			6			7.1			7.2			7.3		
ACTS Changed			40			30			10			70			20,30			40,50,60			50			50			50		
Network Limit			1			1			none			none			none			none									none		
Days Changed			1			1			2			1			1			1									1		
i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag	i	j	Lag			
10	20	0							<<	<																			
10	30	0							<<	<																			
20	40	3					<	2	<	<					<<	<													
20	50	0							<	<					<<	<													
30	40	0				<<	<		<	<					<<	<													
30	60	0				<<	<		<	<					<<	<													
40	70	0	<<	<		<	<		<	<					<	<		<<	<			<			X		X		
50	70	2		<	1		<	0	<	<					<	<		<<	<		<<	<		<<	<		<<	X	1
60	70	1		<	0	<	<		<	<					<	<		<<	<			<					X		

Since lag 40-70 is 0, we cannot move 70 forward as indicated in the table in cycle 7.2. Thus, we will cross ("X") all the arrows for activity 70 out, shown in cycle 7.3.

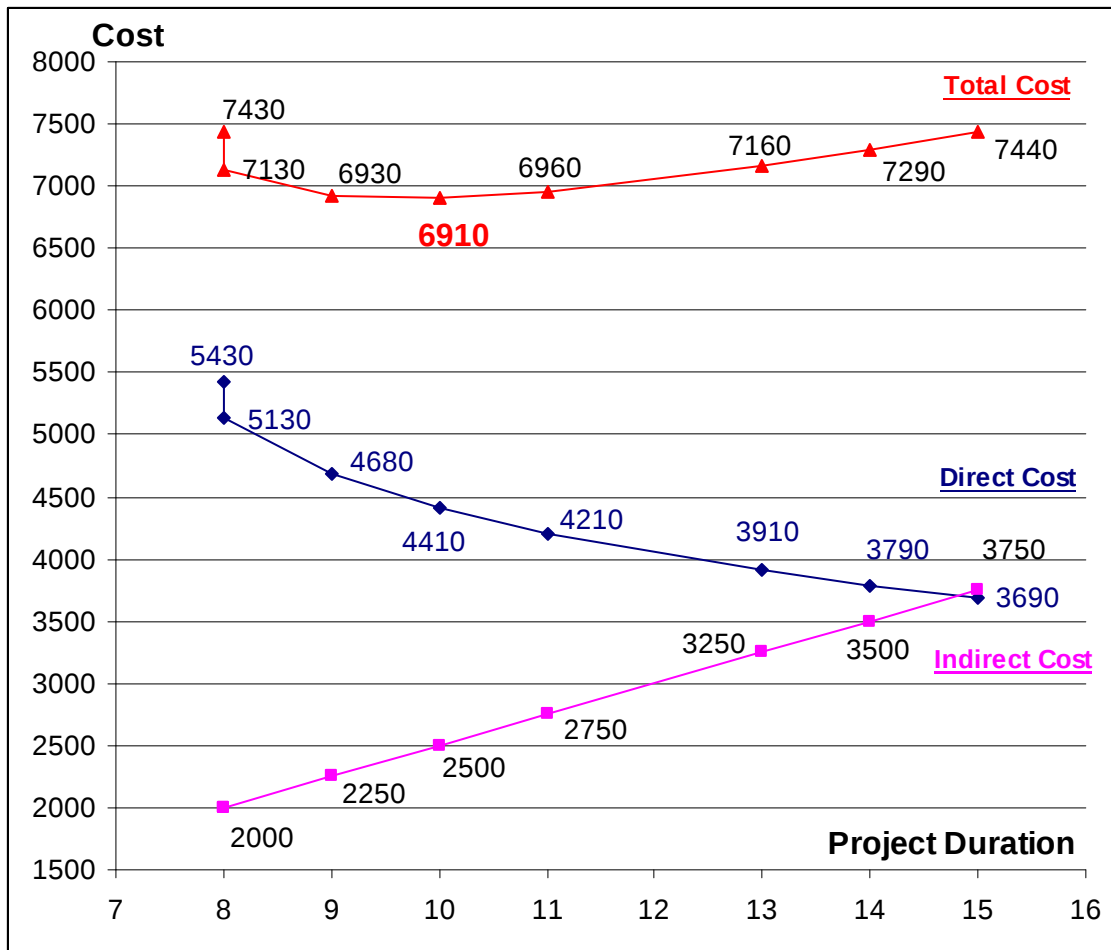
Final schedule at all crash point.

	ID	TC	TN	CC	CN	T
A	10	2	4	600	300	2
B	20	1	2	800	650	1
C	30	3	5	480	240	3
D	40	1	3	300	100	1
E	50	2	4	1600	1000	2
F	60	1	2	1150	1100	1
G	70	2	3	500	300	2
			sum	5430	3690	5430
Project Cost = 5130 + 300 = 5430 at 14 days with 3 critical paths. Act 40 and 60 cannot be crashed anymore.						



Cycle	Duration	Direct Cost	Indirect Cost	Total Cost
	15	3690	3750	7440
1	14	3790	3500	7290
2	13	3910	3250	7160
3	11	4210	2750	6960
4	10	4410	2500	6910
5	9	4680	2250	6930
6	8	5130	2000	7130
7	8	5430	2000	7430

Given indirect cost is \$250/day.



8. TIME-COST TRADEOFF-LP

Example of Time-Cost Tradeoff using LINDO

Formulas: $a = C_n - T_n \cdot b$ $b = (C_n - C_c) / (T_n - T_c)$						
Act No	Normal		Crash		Cost = a + b*Time	
	Time T_n	Cost C_n	Time T_c	Cost C_c	a	b
10	5	500	5	500	500	0
20	3	900	1	1200	1350	-150
30	7	3250	4	4150	5350	-300
32	5	1000	3	1300	1750	-150
40	6	1400	3	2750	4100	-450
42	5	1100	3	1900	3100	-400
44	7	1500	4	2400	3600	-300
50	10	4200	5	7200	10200	-600
52a	6	0	4	500	1500	-250
52b	2	800	0	1000	1000	-100
60	2	1100	1	1300	1500	-200
70	3	1300	2	1500	1900	-200
Normal Cost		17050	All Crash Cost		25700	35850 = Sum of Fixed Costs

Step 1: Calculate COST SLOPE (b) for each activity as shown in the table above.

Step 2: Coding in LINDO (see Figure 2)

Step 2.1: Setup OBJECTIVE FUNCTION in LINDO

MIN -150 T20 - 300 T30 -150 T32 - 450 T40 - 400 T42 - 300 T44 - 600 T50 - 250 T52a - 200 T60 - 100 T52b - 200 T70 + 35850 FIXED

Step 2.2: Tell LINDO, where constraints START by typing in

ST

Step 2.3: Setup PRECEDENCE CONSTRAINTS

T10 + ESD10 - ESD20 <= 0

T20 + ESD20 - ESD30 <= 0

T20 + ESD20 - ESD32 <= 0

T30 + ESD30 - ESD40 <= 0

T30 + ESD30 - ESD42 <= 0

T32 + ESD32 - ESD40 <= 0

$T32 + ESD32 - ESD42 \leq 0$
 $T32 + ESD32 - ESD44 \leq 0$
 $T40 + ESD40 - ESD50 \leq 0$
 $T40 + ESD40 - ESD52a \leq 0$
 $T42 + ESD42 - ESD50 \leq 0$
 $T42 + ESD42 - ESD52a \leq 0$
 $T44 + ESD44 - ESD52a \leq 0$
 $T50 + ESD50 - ESD60 \leq 0$
 $T52a + ESD52a - ESD52b \leq 0$
 $T52b + ESD52b - ESD60 \leq 0$
 $T60 + ESD60 - ESD70 \leq 0$

Step 2.4: Setup ACTIVITY DURATION CONSTRAINTS

$T10 \geq 5$
 $T10 \leq 5$
 $T20 \geq 1$
 $T20 \leq 3$
 $T30 \geq 4$
 $T30 \leq 7$
 $T32 \geq 3$
 $T32 \leq 5$
 $T40 \geq 3$
 $T40 \leq 6$
 $T42 \geq 3$
 $T42 \leq 5$
 $T44 \geq 4$
 $T44 \leq 7$
 $T50 \geq 5$
 $T50 \leq 10$
 $T52a \geq 4$
 $T52a \leq 6$

$$T60 \geq 1$$

$$T60 \leq 2$$

$$T52b \geq 0$$

$$T52b \leq 2$$

$$T70 \geq 2$$

$$T70 \leq 3$$

Step 2.5: Setup PROJECT START DATE

$$ESD10 = 0$$

Step 2.6: Setup TARGETED PROJECT DURATION

$$T70 + ESD70 \leq 36$$

Note that, number of the right hand side should set to NORMAL POINT of project duration, which is 36 days for this example.

Step 2.7: Assign value to FIXED CONSTANT

$$FIXED = 1$$

Note that, because LINDO does not allow constants in objective function, we must create variable (instead of constants) call FIXED in objective function and then assign value of 1 to it later.

Step 2.8: Tell LINDO where constraints END by typing in

END

Step 3.1: Run Solve. (See Figure 3.1)

Step 4.1: After you solve the problem, go to Reports → Parametrics. (See Figure 4.1)

Step 4.2: Setup Parametric Row and New RHS. (See Figure 4.2)

Step 5: Get results from Report Window (Figure 5.1) and Parametrics Graph Window (Figure 5.2)

Begin the new project with blank file.

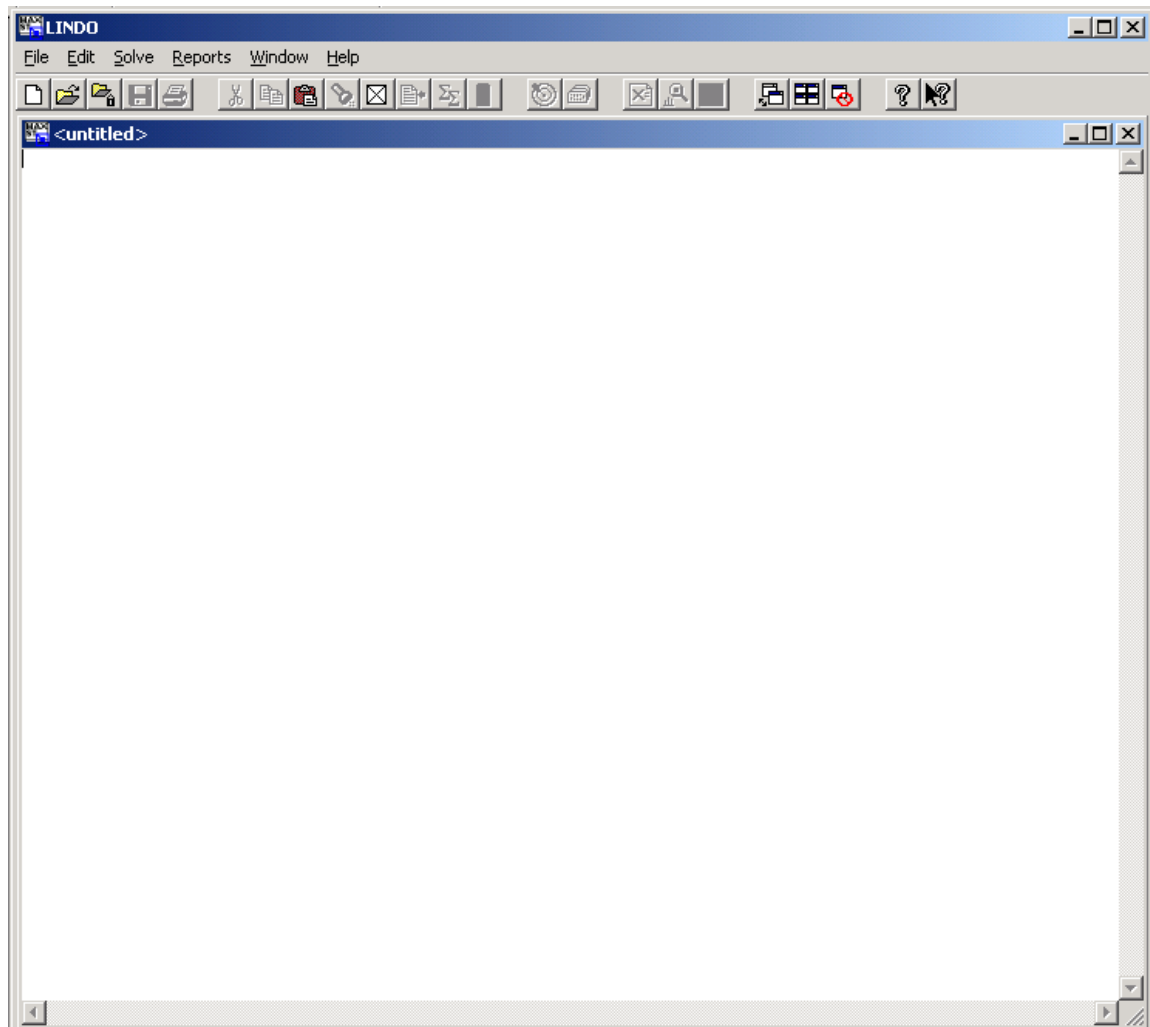


Figure 1: Blank LINDO File.

```

LINDO
File Edit Solve Reports Window Help
C:\Documents and Settings\cem\Desktop\Example for LINDO CEE536 by Chachrist.ltx
MIN -150 T20 - 300 T30 -150 T32 - 450 T40 - 400 T42 - 300 T44 - 600 T50 - 250
ST
T10 + ESD10 - ESD20 <= 0
T20 + ESD20 - ESD30 <= 0
T20 + ESD20 - ESD32 <= 0
T30 + ESD30 - ESD40 <= 0
T30 + ESD30 - ESD42 <= 0
T32 + ESD32 - ESD40 <= 0
T32 + ESD32 - ESD42 <= 0
T32 + ESD32 - ESD44 <= 0
T40 + ESD40 - ESD50 <= 0
T40 + ESD40 - ESD52a <= 0
T42 + ESD42 - ESD50 <= 0
T42 + ESD42 - ESD52a <= 0
T44 + ESD44 - ESD52a <= 0
T50 + ESD50 - ESD60 <= 0
T52a + ESD52a - ESD52b <= 0
T52b + ESD52b - ESD60 <= 0
T60 + ESD60 - ESD70 <= 0
T10 >= 5
T10 <= 5
T20 >= 1
T20 <= 3
T30 >= 4
T30 <= 7
T32 >= 3
T32 <= 5
T40 >= 3
T40 <= 6
T42 >= 3
T42 <= 5
T44 >= 4
T44 <= 7
T50 >= 5
T50 <= 10
T52a >= 4
T52a <= 6
T60 >= 1
T60 <= 2
T52b >= 0
T52b <= 2
T70 >= 2
T70 <= 3
ESD10 = 0
T70 + ESD70 <= 36
FIXED = 1
END

```

Figure 2: Coding.

Note that, the highlighted row is called “Parametric Row”, which will be used to derive minimum project cost at different project duration. Each equation (objective function and constraints) equals to 1 row, excluding ST and END commands. So, the Parametric Row in Figure 2 is 44.

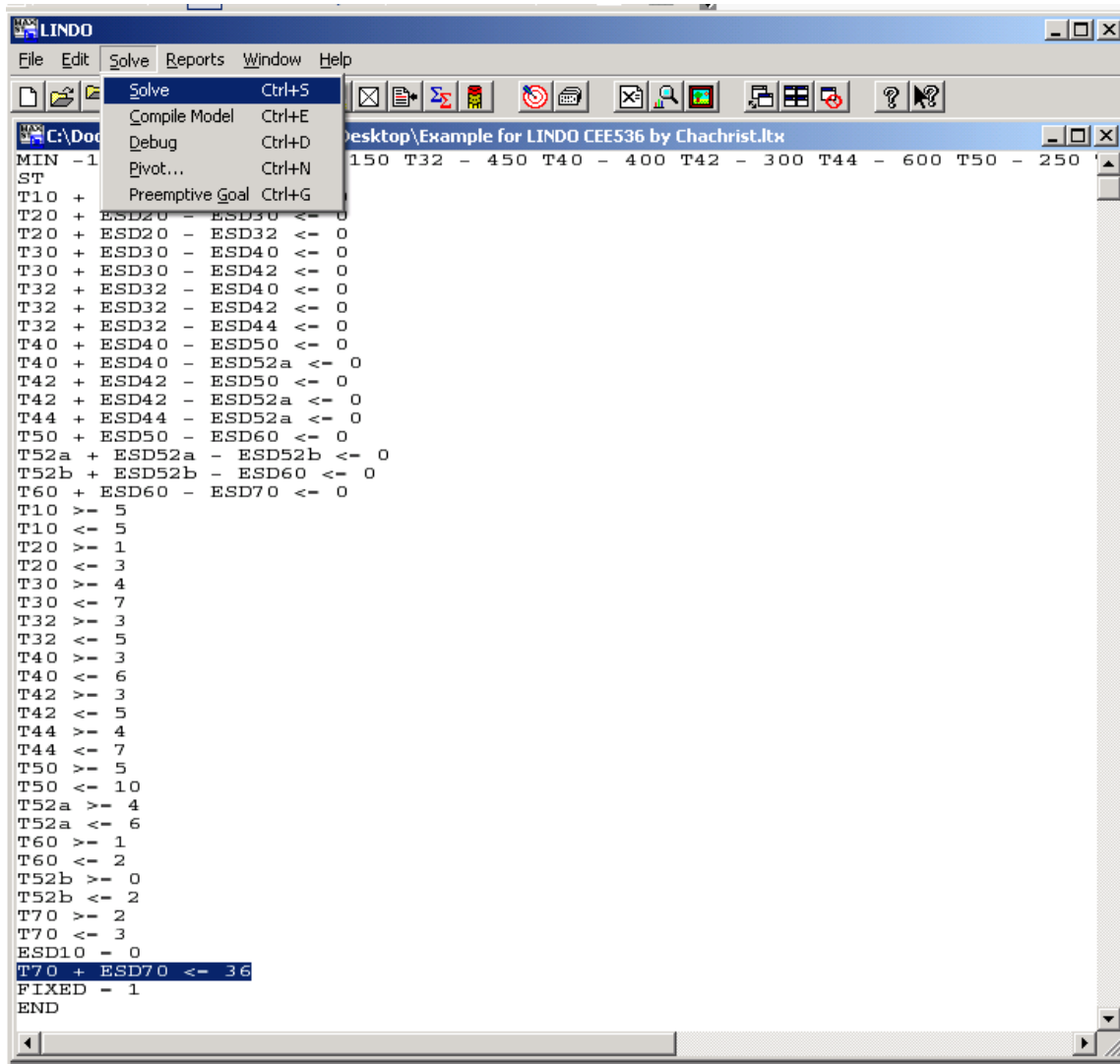


Figure 3.1: Solving

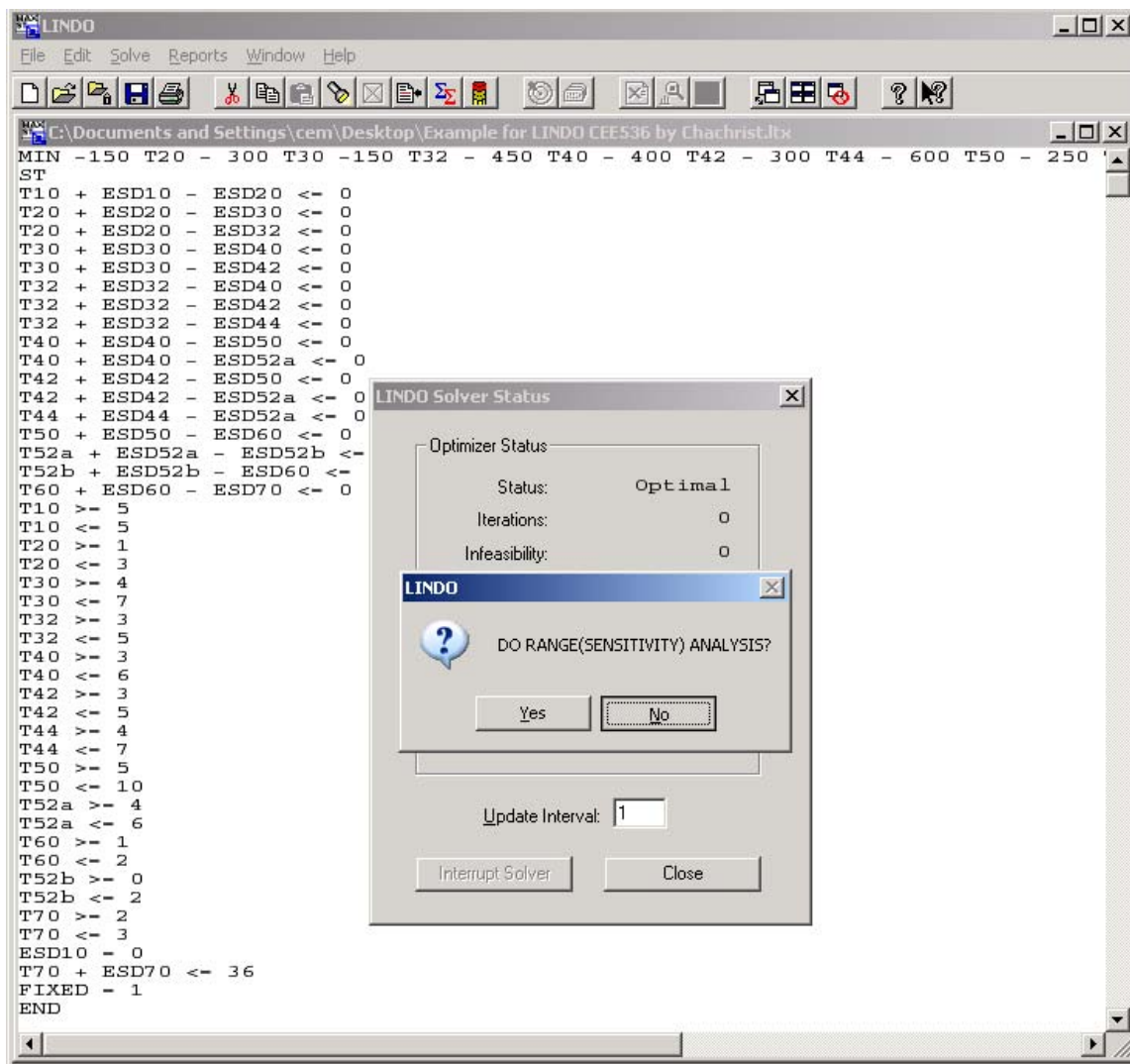


Figure 3.2: After solving the problem , click No, and the Close.

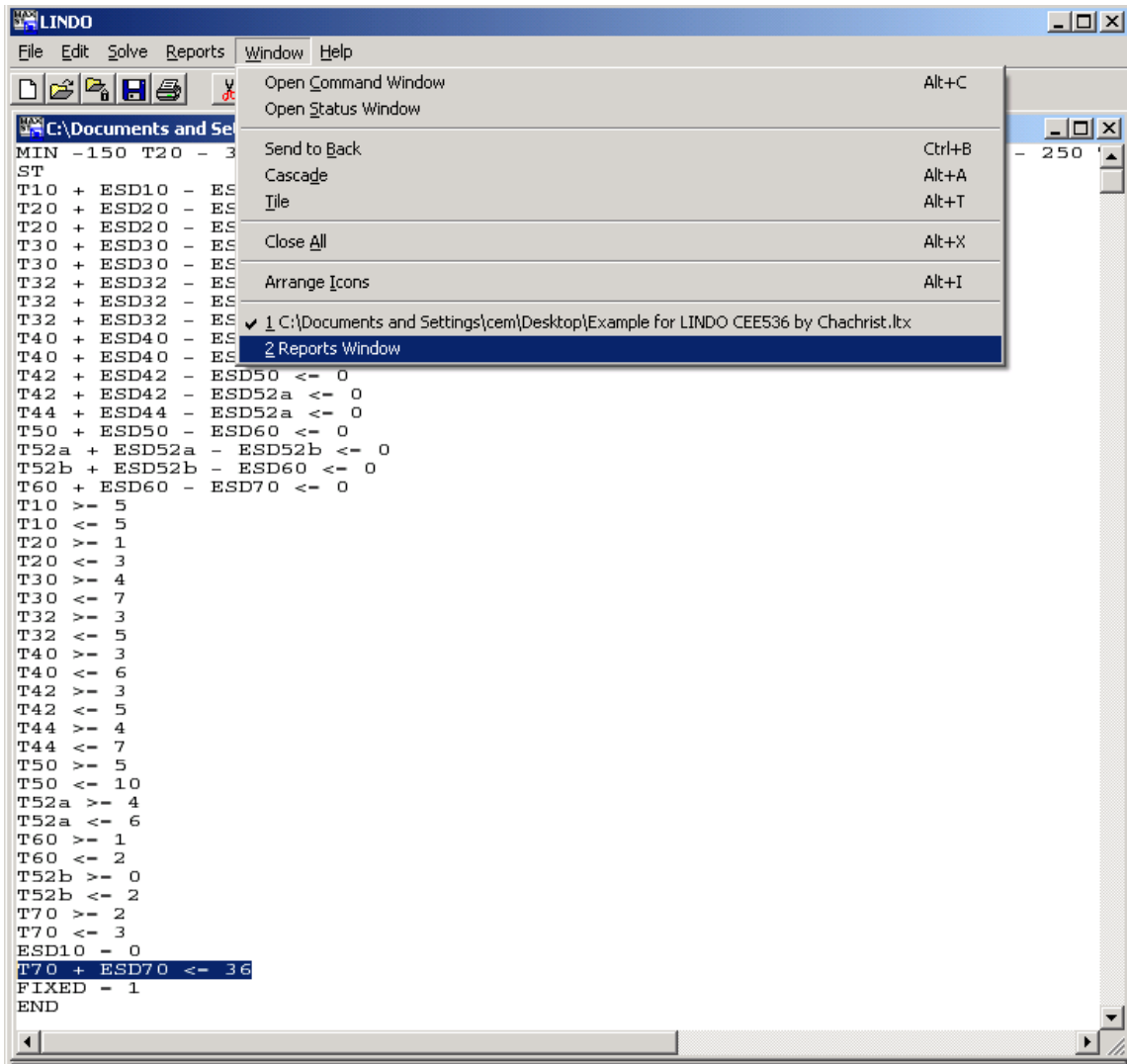


Figure 3.3: To check the results from solving (or minimizing) the problem under the condition of project duration must equal to 36 days based on the highlighted code.

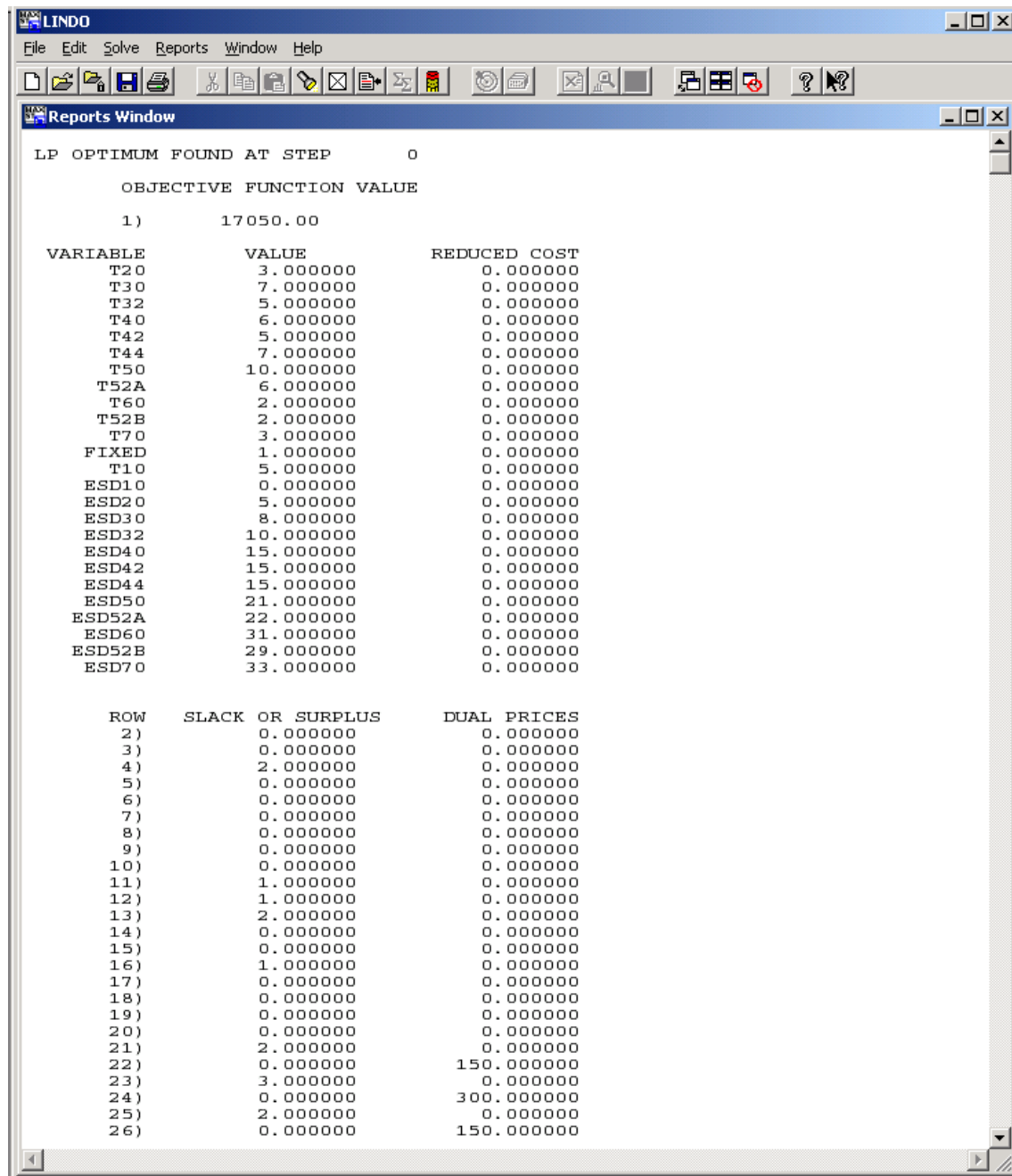


Figure 3.4: Results from solving (or minimizing) the problem at specified project duration of 36 days.

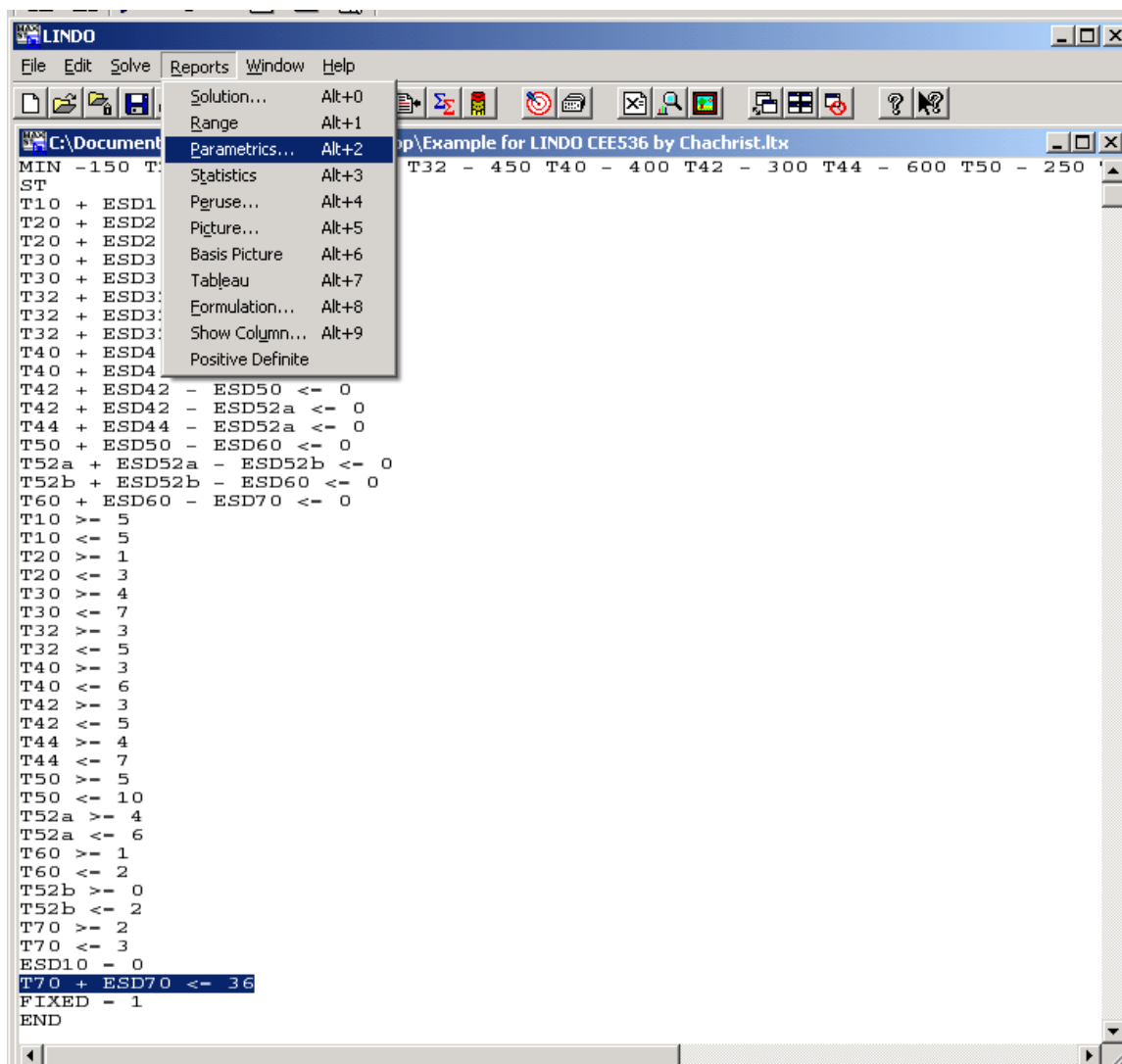


Figure 4.1: Parametrics

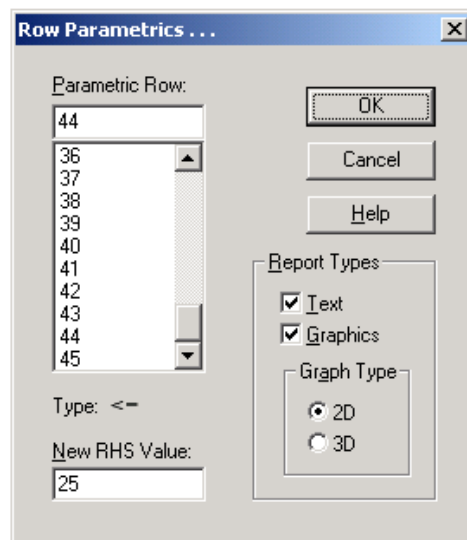


Figure 4.2: Setup Parametric options.

We want LINDO to automatically change the project duration and find the minimum project cost. To do so, set New RHS Value to CRASH POINT of project duration. (RHS = Right Hand Side). LINDO will change the number on the RHS of the code in the specified Parametric Row (highlighted code in Figure 2)

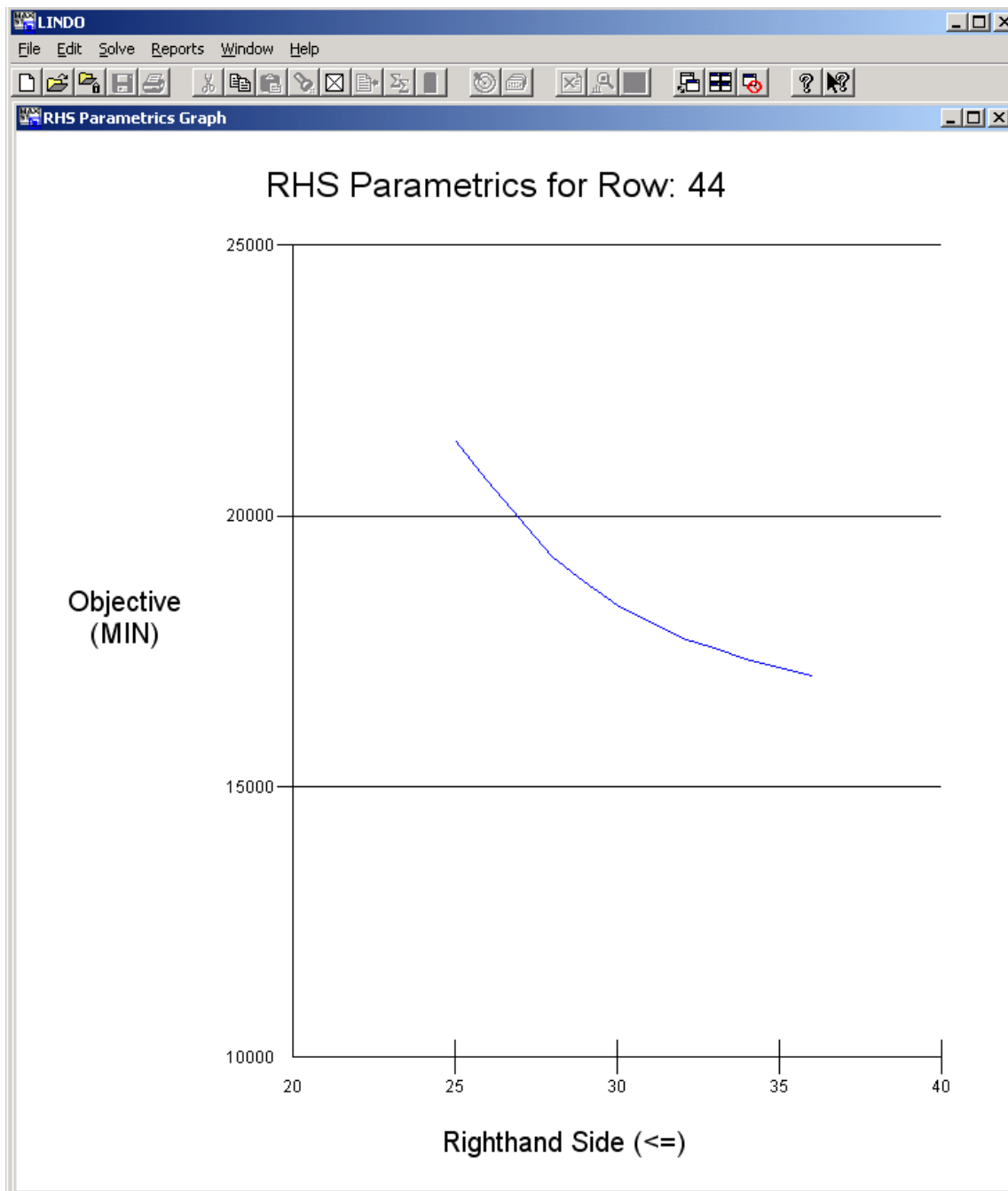
Make sure your Parametric Row is correct. Each equation equals to 1 row, excluding ST and END commands.

RIGHTHANDSIDE PARAMETRICS REPORT FOR ROW: 44

VAR OUT	VAR IN	PIVOT ROW	RHS VAL	DUAL PRICE BEFORE PIVOT	OBJ VAL
SLK 5	SLK 20	5	36.0000	0.000000E+00	17050.0
SLK 19	SLK 22	19	36.0000	0.000000E+00	17050.0
SLK 21	SLK 38	21	34.0000	150.000	17350.0
SLK 37	SLK 42	37	33.0000	200.000	17550.0
SLK 41	SLK 24	41	32.0000	200.000	17750.0
SLK 4	SLK 26	24	30.0000	300.000	18350.0
SLK 23	SLK 28	23	29.0000	450.000	18800.0
SLK 12	SLK 34	44	28.0000	450.000	19250.0
SLK 16	SLK 40	34	28.0000	600.000	19250.0
T52B	SLK 7	40	26.0000	700.000	20650.0
SLK 25	SLK 36	25	25.0000	750.000	21400.0

Figure 5.1: Results from Report Window.

Notice that, LINDO optimized project cost from 36 days (our original value or NORMAL POINT) to 25 days (our New RHS Value or CRASH POINT)



Using Excel Solver to solve Time-Cost Tradeoff problem

Microsoft Excel - Set Solver for time cost trade off in excel

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2																			
3																			
4																			
5																			
6																			
7																			
8																			
9																			
10																			
11																			
12																			
13																			
14																			
15																			
16																			
17																			
18																			
19																			

Step one: create a table from given data

	NO	TN	CN	TC	CC
_10	10	5	500	5	500
_20	20	3	900	1	1200
_30	30	7	3250	4	4150
_32	32	5	1000	3	1300
_40	40	6	1400	3	2750
_42	42	5	1100	3	1900
_44	44	7	1500	4	2400
_50	50	10	4200	5	7200
_52.1	52.1	6	0	4	500
_52.2	52.2	2	800	0	1000
_60	60	2	1100	1	1300
_70	70	3	1300	2	1500

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Step two: Input formula for "v", "C", and FD

$=IF(D6-F6<>0,(E6-G6)/(D6-F6),0)$

$=(D6-I6)*H6+E6$

$=K6+I6$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2																			
3																			
4																			
5			NO	TN	CN	TC	CC	v	T	C	SD	FD							
6		_10	10	5	500	5	500	0		500		0							
7		_20	20	3	900	1	1200	-150		1350		0							
8		_30	30	7	3250	4	4150	-300		5350		0							
9		_32	32	5	1000	3	1300	-150		1750		0							
10		_40	40	6	1400	3	2750	-450		4100		0							
11		_42	42	5	1100	3	1900	-400		3100		0							
12		_44	44	7	1500	4	2400	-300		3600		0							
13		_50	50	10	4200	5	7200	-600		10200		0							
14		_52.1	52.1	6	0	4	500	-250		1500		0							
15		_52.2	52.2	2	800	0	1000	-100		1000		0							
16		_60	60	2	1100	1	1300	-200		1500		0							
17		_70	70	3	1300	2	1500	-200		1900		0							
18																			
19																			

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L17 $\text{fx} = \text{K17} + \text{I17}$

Step three : Create NAME for table by cropping the table including row and column header and then go to Insert, Name, Create.

	NO	TN	CN	TC	CC	v	T	C	SD	FD
_10	10	5	500	5	500	0		500		0
_20	20	3	900	1	1200	-150		1350		0
_30	30	7	3250	4	4150	-300		5350		0
_32	32	5	1000	3	1300	-150		1750		0
_40	40	6	1400	3	2750	-450		4100		0
_42	42	5	1100	3	1900	-400		3100		0
_44	44	7	1500	4	2400	-300		3600		0
_50	50	10	4200	5	7200	-600		10200		0
_52.1	52.1	6	0	4	500	-250		1500		0
_52.2	52.2	2	800	0	1000	-100		1000		0
_60	60	2	1100	1	1300	-200		1500		0
_70	70	3	1300	2	1500	-200		1900		0

Create Names

Create names in

- ☒ Top row
- ☒ Left column
- ☐ Bottom row
- ☐ Right column

OK Cancel

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N5 Node i

Step four : Create the second table by repeating step 1 to 3, only this time set formula for "Lag".
(Don't forget to crop the entire table including column header in order to set name variable for "Lag")

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
2																		
3																		
4																		
5			NO	TN	CN	TC	CC	v	T	C	SD	FD		Node i	Node j	Lag		
6		_10	10	5	500	5	500	0			500		0	10	20	0	= SD_20-SD_10-T_10	
7		_20	20	3	900	1	1200	-150			1350		0	20	30	0	= SD_30-SD_20-T_20	
8		_30	30	7	3250	4	4150	-300			5350		0	20	32	0	= SD_32-SD_20-T_20	
9		_32	32	5	1000	3	1300	-150			1750		0	30	40	0	= SD_40-SD_30-T_30	
10		_40	40	6	1400	3	2750	-450			4100		0	30	42	0	= SD_42-SD_30-T_30	
11		_42	42	5	1100	3	1900	-400			3100		0	32	40	0	= SD_40-SD_32-T_32	
12		_44	44	7	1500	4	2400	-300			3600		0	32	42	0	= SD_42-SD_32-T_32	
13		_50	50	10	4200	5	7200	-600			10200		0	32	44	0	= SD_44-SD_32-T_32	
14		_52.1	52.1	6	0	4	500	-250			1500		0	40	50	0	= SD_50-SD_40-T_40	
15		_52.2	52.2	2	800	0	1000	-100			1000		0	40	52.1	0	= SD_52.1-SD_40-T_40	
16		_60	60	2	1100	1	1300	-200			1500		0	42	50	0	= SD_50-SD_42-T_42	
17		_70	70	3	1300	2	1500	-200			1900		0	42	52.1	0	= SD_52.1-SD_42-T_42	
18														44	52.1	0	= SD_52.1-SD_44-T_44	
19														50	60	0	= SD_60-SD_50-T_50	
20														52.1	52.2	0	= SD_52.2-SD_52.1-T_52	
21	Note :													52.2	60	0	= SD_60-SD_52.2-T_52.2	
22														60	70	0	= SD_70-SD_60-T_60	
23																		
24																		
25																		
26																		
27																		
28																		
29																		
30																		
31																		
32																		

In order to refer to value by using row and column header we must type column name follow by SPACE and then row header. If NAME is set correctly during input column and row header letter in input formula box will change from black to green or blue.

For example : =SD_20-SD_10 - T_10

↑ ↑ ↑

SPACE!!

Microsoft Excel - Set Solver for time cost trade off in excel

File Edit View Insert Format Tools Data Window Help Adobe PDF

Type a question for help

80%

Security...

J18 0

Step five : Insert formula for "ProjCost". Then create variable "ProjCost", "FDproj", and "TS" by cropping the cell and go to Insert, Define, and input the desired name for the variable.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2																			
3																			
4																			
5			NO	TN	CN	TC	CC	v	T	C	SD	FD		Node i	Node j	Lag			
6			_10	10	5	500	5	500	0	500		0		10	20	0			
7			_20	20	3	900	1	1200	-150	1350		0		20	30	0			
8			_30	30	7	3250	4	4150	-300	5350		0		20	32	0			
9			_32	32	5	1000	3	1300	-150	1750		0		30	40	0			
10										4100		0		30	42	0			
11										3100		0		32	40	0			
12										3600		0		32	42	0			
13										10200		0		32	44	0			
14										1500		0		40	50	0			
15										1000		0		40	52.1	0			
16										1500		0		42	50	0			
17										1900		0	FDproj	42	52.1	0			
18									ProjCost	0	TS	28		44	52.1	0			
19														50	60	0			
20														52.1	52.2	0			
21														52.2	60	0			
22														60	70	0			
23																			
24																			
25																			
26																			

Define Name

Names in workbook:

ProjCost

OK

Close

Add

Delete

Refers to:

=step5!\$J\$18

This cell is "ProjCost"

Formula in this cell is =SUM(C_)

This cell is "TS"

Just input reasonable number

Microsoft Excel - Book1

File Edit View Insert Format Tools Data Window Help Adobe PDF

Type a question for help

67%

Security...

B3 Step six : Go to Tools, Add-ins, and click on Solver Add-in.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1																						
2																						
3																						
4																						
5			ND	TN	CN	TC	CC	v	7													
6		_10	10	5	500	5	500	0														
7		_20	20	3	900	1	1200	-150														
8		_30	30	7	3250	4	4150	-300														
9		_32	32	5	1000	3	1300	-150														
10		_40	40	6	1400	3	2750	-450														
11		_42	42	5	1100	3	1900	-400														
12		_44	44	7	1500	4	2400	-300														
13		_50	50	10	4200	5	7200	-600														
14		_52.1	52.1	6	0	4	500	-250														
15		_52.2	52.2	2	800	0	1000	-100														
16		_60	60	2	1100	1	1300	-200														
17		_70	70	3	1300	2	1500	-200														
18									ProjCo													
19																						
20																						
21																						
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30																						
31																						
32																						

Step six : Go to Tools, Add-ins, and click on Solver Add-in.

Add-Ins

Add-Ins available:

- ☐ Analysis ToolPak
- ☐ Analysis ToolPak - VBA
- ☐ Conditional Sum Wizard
- ☐ Euro Currency Tools
- ☐ Internet Assistant VBA
- ☐ Lookup Wizard
- ☒ Solver Add-in

OK

Cancel

Browse...

Automation...

Solver Add-in

Tool for optimization and equation solving

Microsoft Excel - Book1

File Edit View Insert Format Tools Data Window Help Adobe PDF

Type a question for help

67%

ProjCost =SUM(C_)

Step seven : Go to Tools, Solver. And input target cell and constraints. Don't forget to click Min.

	NO	TN	CN	TC	CC	v	f	C	SD	FD	Node i	Node j	Lag
10	10	5	500	5	500	0	5	500	0	5	10	20	0
20	20	3	900	1	1200	-150	1	1200	5	6	20	30	0
30	30	7	3250	4	4150	-300	4	4150	6	10	20	32	0
32	32	5	1000	3	1300	-150	4	1150	6	10	30	40	0
40	40	6	1400	3	2750	-450	5	1850	10	15	30	42	0
42	42	5	1100	3	1900	-400	5	1100	10	15	32	42	0
44	44	7	1500	4	2400	-300	7	1500	10	17	32	42	0
50	50	10	4200	5	7200	-600	10	4200	15	25	32	42	0
52.1	52.1	6	0	4	500	-250	6	0	17	23	40	42	0
52.2	52.2	2	800	0	1000	-100	2	800	23	25	40	42	0
60	60	2	1100	1	1300	-200	1	1300	25	26	42	42	0
70	70	3	1300	2	1500	-200	2	1500	26	28	42	42	0

ProjCost 192500 TS 28

FDproj 42

50

52.1

52.2

60

Solver Parameters

Set Target Cell: ProjCost

Equal To: ☐ Max ☒ Min ☐ Value of: 0

By Changing Variable Cells: T,SD

Subject to the Constraints:

- \$K\$6 = 0
- FDproj <= TS
- Lag >= 0
- T <= TN
- T >= TC

Solver Options

Max Time: 100 seconds

Iterations: 100

Precision: 0.000001

Tolerance: 5 %

Convergence: 0.0001

☒ Assume Linear Model ☐ Use Automatic Scaling

☐ Assume Non-Negative ☐ Show Iteration Results

Estimates **Derivatives** **Search**

- ☒ Tangent ☒ Forward ☒ Newton
- ☐ Quadratic ☐ Central ☐ Conjugate

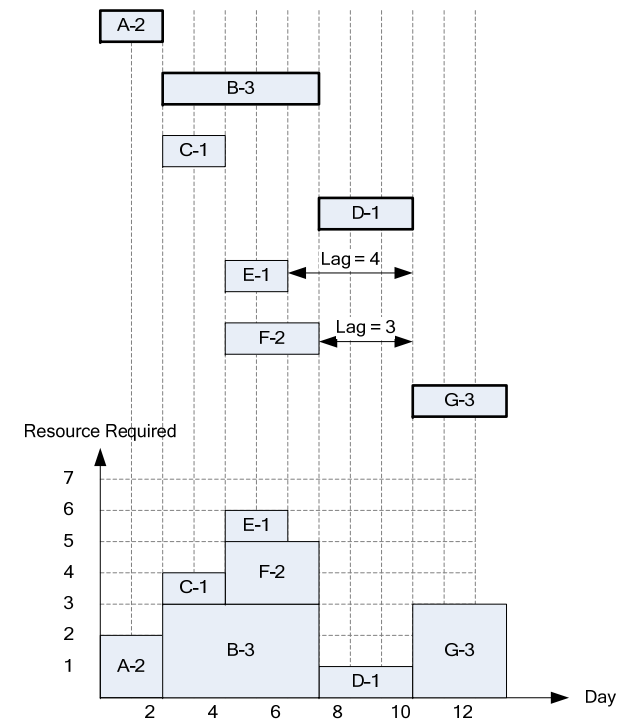
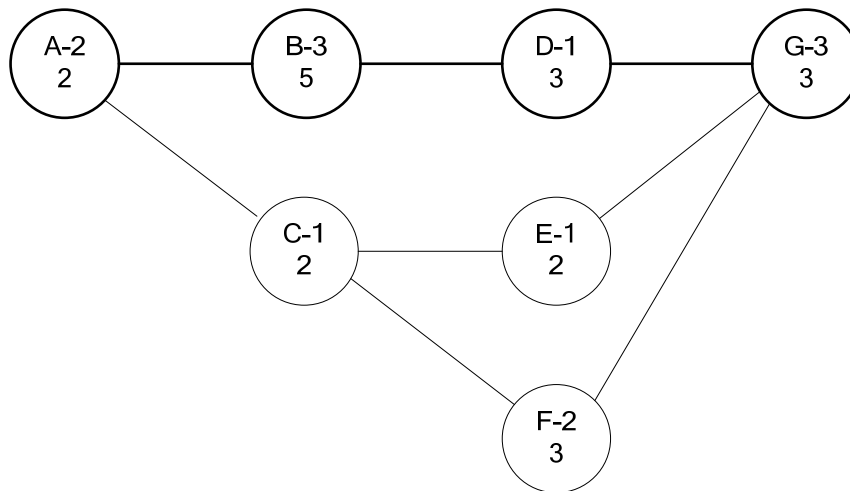
Step seven : Go to Tools, Solver. And input target cell and constraints. Don't forget to click Min. Then click Options and click on Assume Linear Model....FINALLY click Solve

9. RESOUCÉ LEVELING

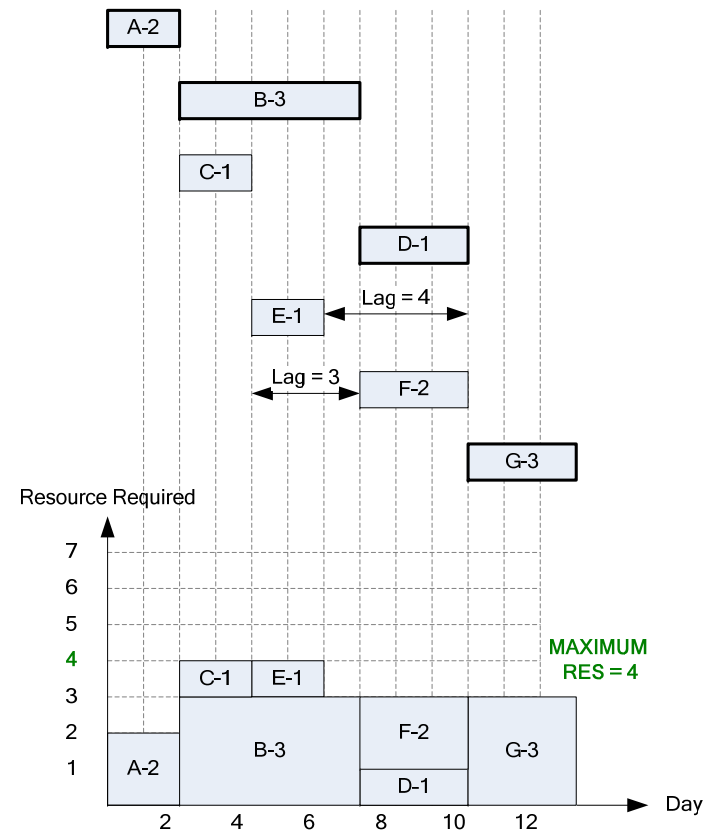
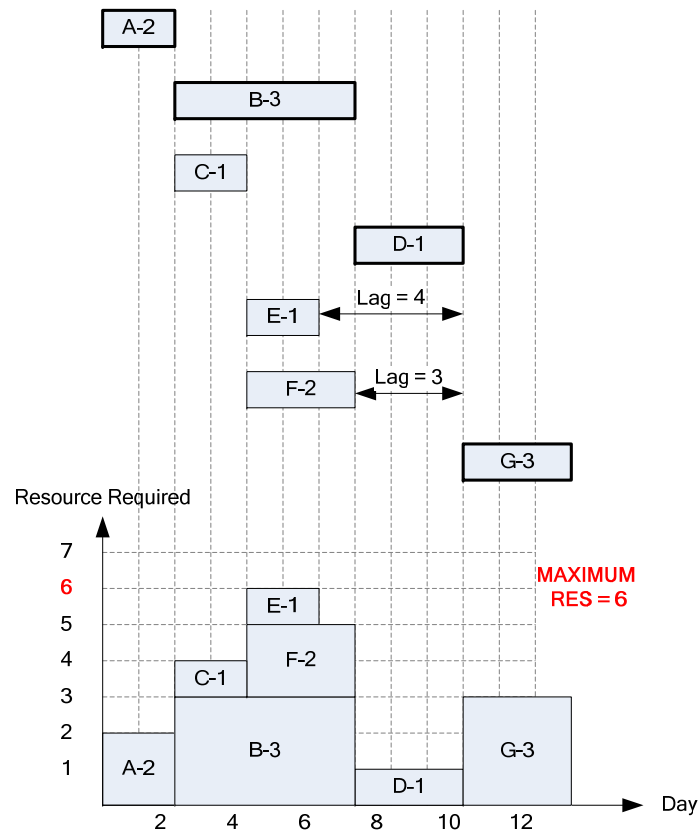
- MINIMUM MOMENT METHOD

Resource Leveling

In this section, the two concepts of resource leveling, Minimum Moment Method and PACK, are discussed. First let's take a look at the network diagram below consisting of 7 activities. As shown in the bar chart and resource profile, if activities are scheduled at their early start date (ESD), this project requires the MAXIMUM RESOURCE of SIX on day 5 and 6. On the other hand, this schedule will use only ONE resource on day 8, 9, and 10. This fluctuation of resource profile may result in difficulty in managing resource. Consequently, it adversely affects the project.



Now, look at the new resource profile after it is leveled. As you can see, maximum resource required reduces from 6 to 4 without increasing project duration. You may wonder what if we apply limited resource allocation with maximum resource available of 4. Well, hopefully the next 3 questions and answers will make my point clearer.



Question 1: So, what is the main focus here?

Answer 1.1: Reduce fluctuation in resource profile, while project duration remains the same. ☺

Question 2: Why don't just use Limited Resource Allocation?

Answer 2.1: Limited resource allocation doesn't directly solve the problem of fluctuation. The maximum resource required may change but the resource profile could STILL be fluctuated.

Answer 2.2: Applying limited resource allocation might increase project duration. We don't want to delay the project!!

Answer 2.3: We may not really have "the maximum number of available resource" yet.

Question 3: So! What do we do?

Answer 3: Minimum Moment Method (MOM) or PACK.

One last time, I want to point out the followings

- 1) Unlimited resource leveling reduces fluctuation in resource profile.
- 2) Unlimited resource leveling never delays critical activities.
- 3) Accordingly, unlimited resource leveling never increases project duration.

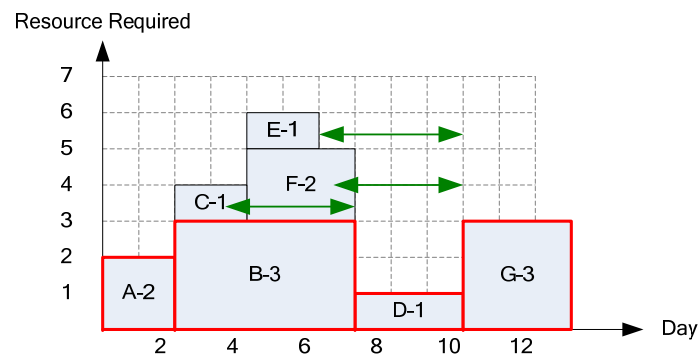
Before going into detail, I want to give you a big picture of these 2 resource leveling methods.

PRIORITY OF NON-CRITICAL ACTIVITIES

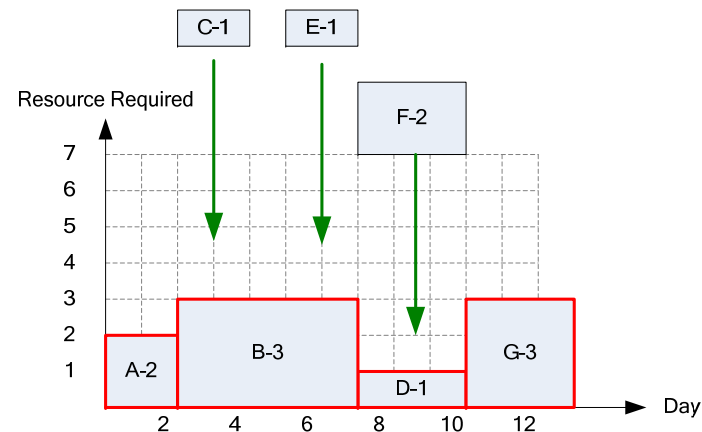
In MOM, calculation starts from the LAST sequence step. This means non-critical activities in the LATER sequence step have higher priority than those in the EARLIER sequence step. Then, what is called “Improvement Factor” is used to prioritize non-critical activities in the same sequence step.

In PACK, it just goes right at the problem. It takes all the non-critical activities out of resource profile (this method is hardcore). And then, place them one by one in the order of MAX to MIN number of resource required. Activity requiring more resource has higher priority than activity needs less resource (this is actually a smart way). As shown in PACK picture, non-critical activities are removed from resource profile. Then, they are positioned one by one at a time.

Minimum Moment Method (MOM)



PACK



MEASUREMENT OF RESOURCE PROFILE

For both methods, a parameter called RIC is used in order to determine the **OVERALL** improvement in resource profile after leveling.

$$RIC = \frac{n \sum R^2}{(\sum R)^2}$$

n = project duration

R = Sum of resource required for a particular day

More about RIC can be found in Precedence and Arrow Networking Techniques for Construction by Robert B. Harris

Minimum Moment Method (MOM)

MOM uses a factor calls IMPROVEMENT FACTOR (IF) to measure the effectiveness of scheduling non-critical activity at different positions. **The greater the IF, the better it is.**

$$IF(Act, S) = r(X - W - mr)$$

S = Number of shifting day (start from 1 to the minimum current lag of between the activity and its successors)

r = Number of resources required by the activity

X = Sum of resources in the range of the activity BEFORE shifting

W = Sum of resources in the range of the activity AFTER shifting

m = The minimum between S and T (duration of the activity).

See! The formula is very simple☺. Now let's solve the previous problem using MOM.

Here is 2 simple rules about shifting activity according to derived IF.

- 1) Negative value of IF is neglected. NO shifting.

2) Only shifting ONE activity with highest IF, one at a time.

(NEVER SHIFT 2 ACTIVITIES IN THE SAME ROUND)

3) If there is a tie of IF in one activity, shifting the activity to the higher S.

For example, $IF(M,2) = 12$ and $IF(M,4) = 12$

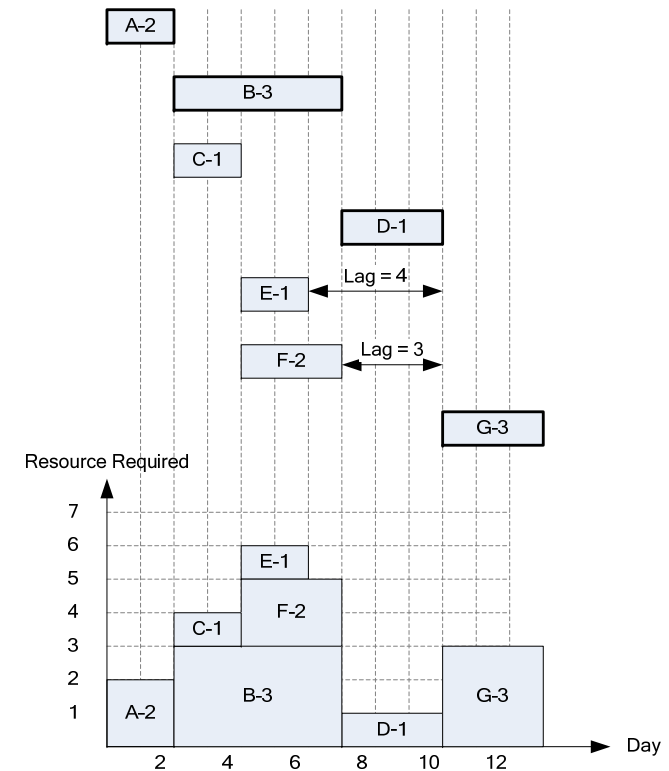
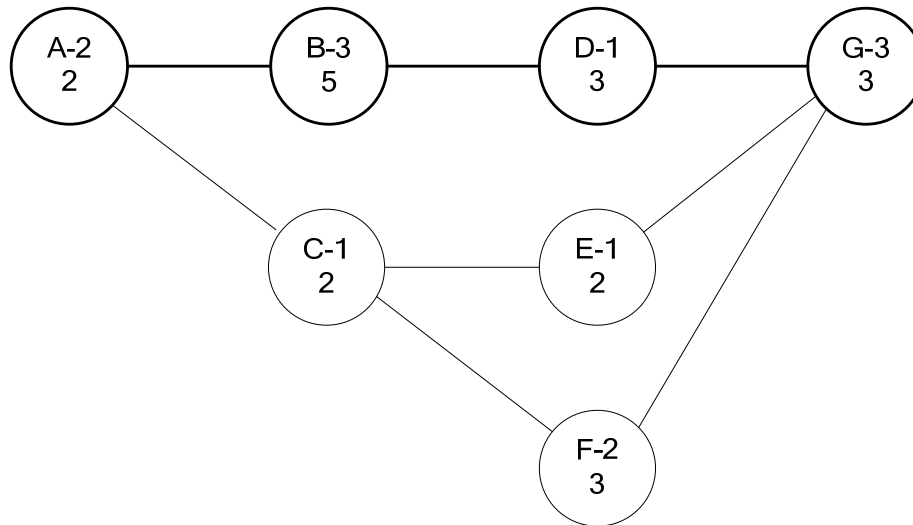
Act M will be shifted by 4 days to give room for its predecessors (make sense, right)

4) If there is a tie of IF between two activities, shifting the activity with the higher resource require.

5) If there is more than one activity in a SQS, you probably have to recalculate IF for the 2nd round, AT LEAST.

Example on Minimum Moment Method (MOM)

Please level the below project using MOM.



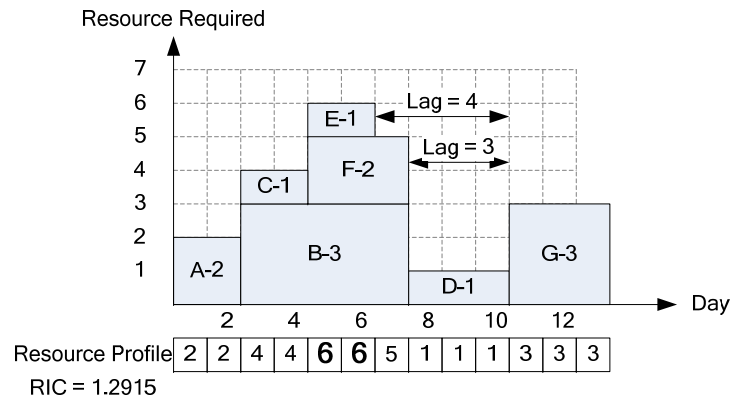
Sequence Step 4 Round 1.

First, we start from the last sequence step. However, since there is no non-critical activity in SQS 4, we move on SQS 3.

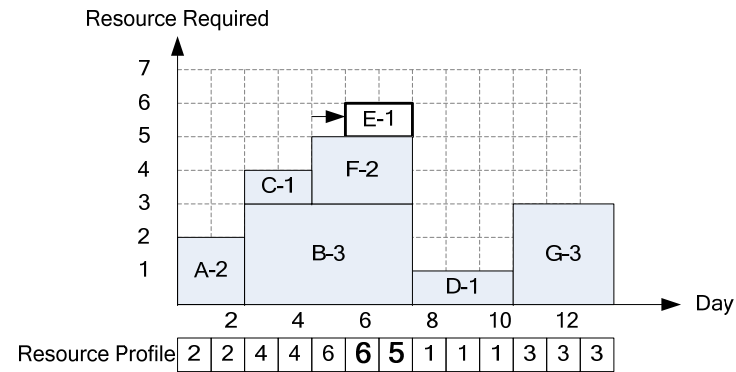
Sequence Step 3 Round 1.

FORWARD PASS: Showing the calculation of X and W for Act E in SQS 3 Round 1

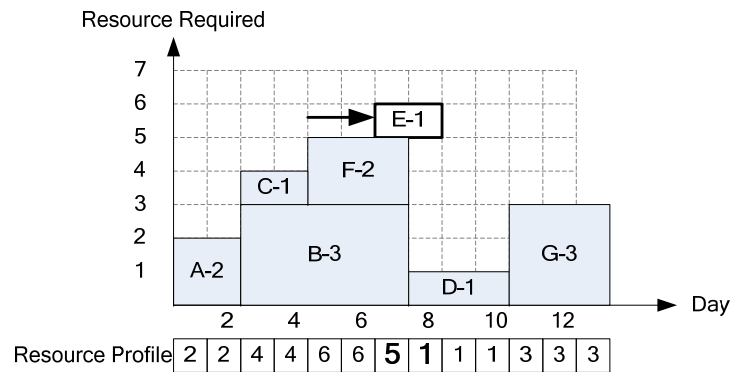
X = Sum of Resources in the range of
Act E **BEFORE** shifting = $6+6 = 12$



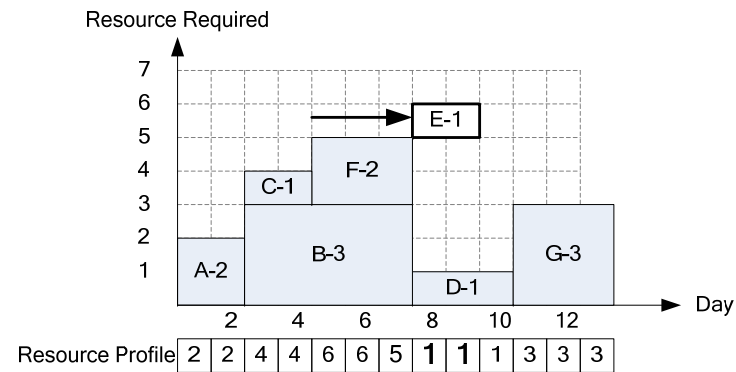
W of 1 shifting day = Sum of Resources in the
range of Act E **AFTER** shifting by 1 day = $6+5 = 11$



W of 2 shifting days = Sum of Resources in the
range of Act E **AFTER** shifting by 2 days = $5+1 = 6$

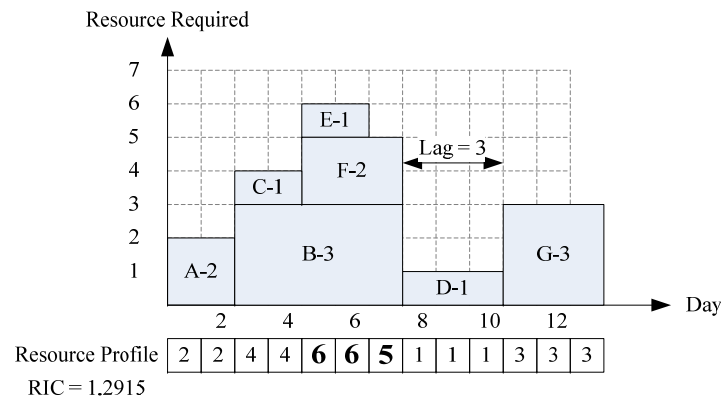


W of 3 shifting days = Sum of Resources in the
range of Act E **AFTER** shifting by 3 days = $1+1 = 2$

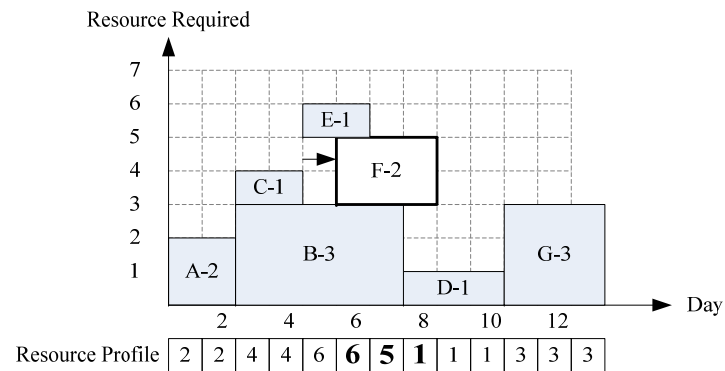


FORWARD PASS: Showing the calculation of X and W for Act F in SQS 3 Round 1

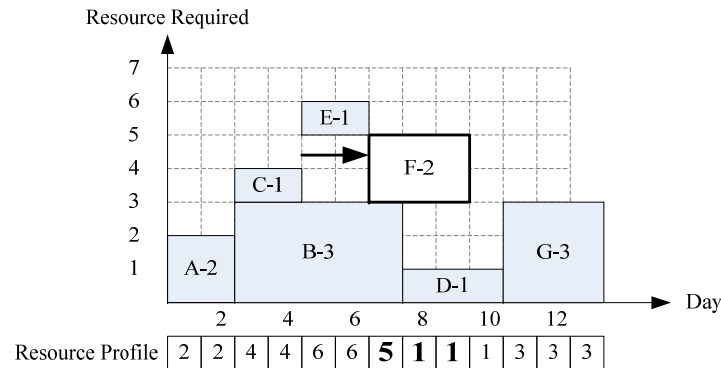
X = Sum of Resources in the range of Act F
BEFORE shifting = $6+6+5 = 17$



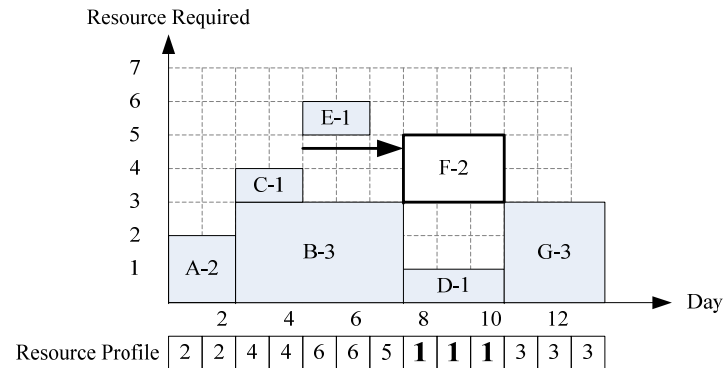
W of **1 shifting day** = Sum of Resources in the range
of Act F **AFTER shifting by 1 day** = $6+5+1 = 12$



W of **2 shifting days** = Sum of Resources in the range
of Act F **AFTER shifting by 2 days** = $5+1+1 = 7$



W of **3 shifting days** = Sum of Resources in the range
of Act F **AFTER shifting by 3 days** = $1+1+1 = 3$



$$IF(Act, S) = r(X - W - mr)$$

Act	S	r	X	W	m = min(S,T)	IF
E	1	1	6+6 = 12	6+5 = 11	1	0
E	2	1	12	5+1= 6	2	4
E	3	1	12	1+1 = 2	2	8
E	4	1	12	1+1 =2	2	8
F	1	2	6+6+5=17	6+5+1=12	1	6
F	2	2	17	5+1+1=7	2	12
F	3	2	17	1+1+1=3	3	16

According to the IF from the above table, Act F is shifted by 3 days, since it provides the max IF.

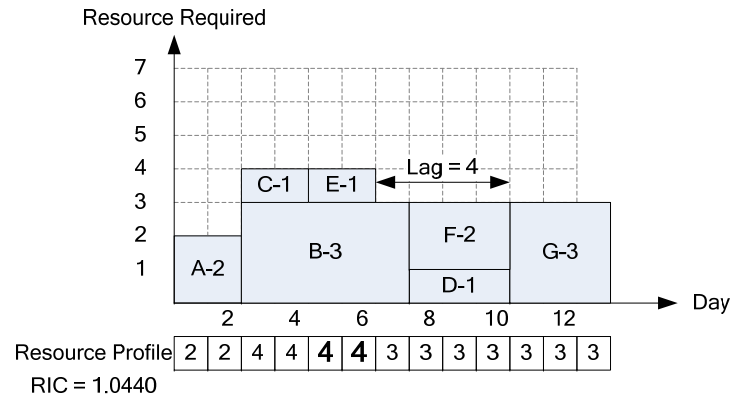
Note: IF of shifting E by 4 days equals to shifting it by 3 days. So, I didn't show it here. ☺

Sequence Step 3 Round 2. Since shifting F changes resource profile within the range of Act E, it is necessary to recalculate IF of E again. And see if there is positive value of IF. **Don't forget to update the resource profile after shifting F!!**

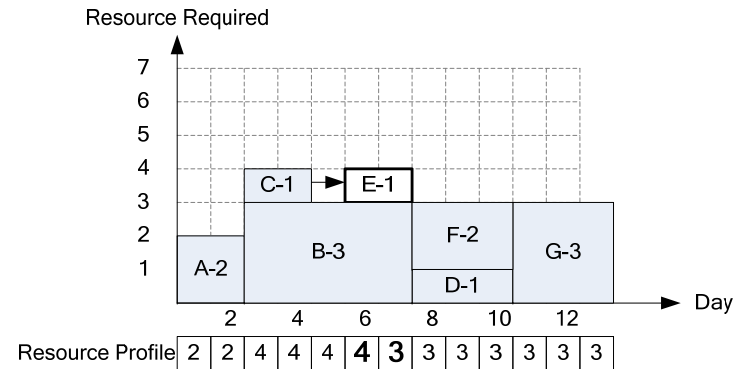
FORWARD PASS: Showing the calculation of X and W for Act

E in SQS 3 Round 2

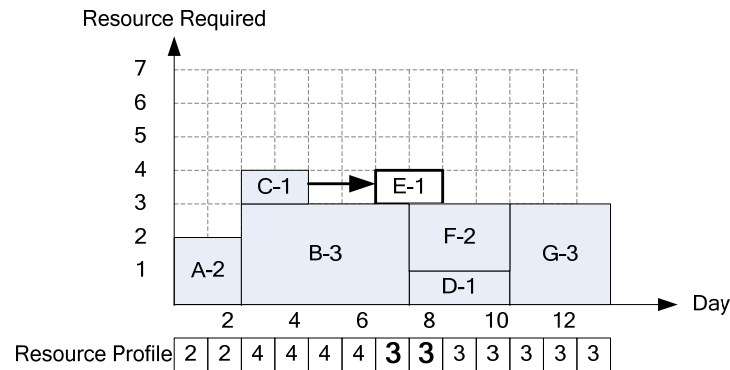
X = Sum of Resources in the range of Act E **BEFORE** shifting = 4+4 = 8



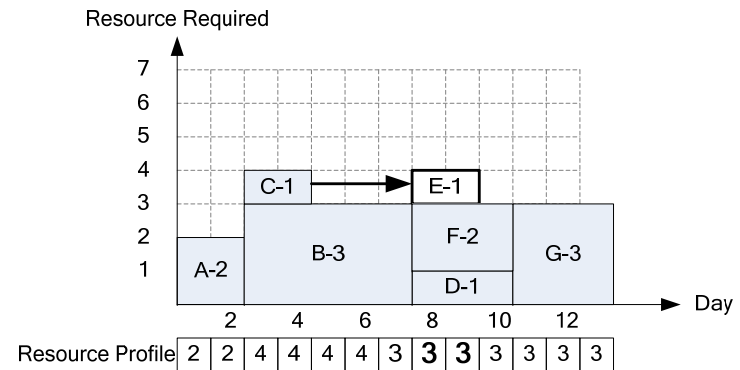
W of 1 shifting day = Sum of Resources in the range of Act E **AFTER** shifting by 1 day = 4+3 = 7



W of 2 shifting days = Sum of Resources in the range of Act E **AFTER** shifting by 2 days = 3+3 = 6



W of 3 shifting days = Sum of Resources in the range of Act E **AFTER** shifting by 3 days = 3+3 = 6



$$IF(Act, S) = r(X - W - mr)$$

Act	S	r	X	W	$m = \min(S,T)$	IF
E	1	1	$4+4 = 8$	$4+3 = 7$	1	0
E	2	1	8	$3+3 = 6$	2	0
E	3	1	8	$3+3 = 6$	2	0
E	4	1	8	$3+3 = 6$	2	0

Thus in SQS 3 Round 2, activity E is shifted by 4 days. **Why not shifting E by either 2 or 3 days since they have the same IF????**

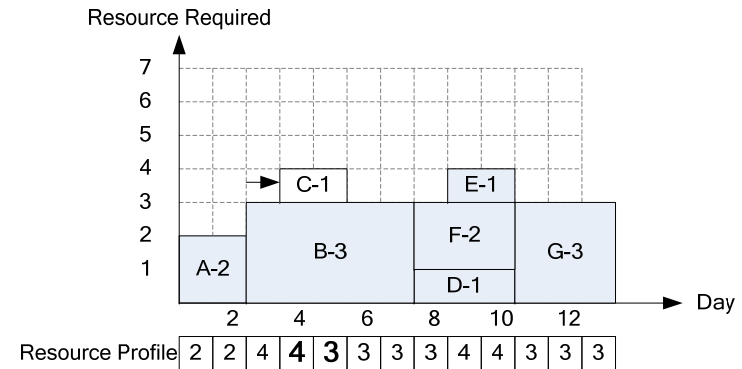
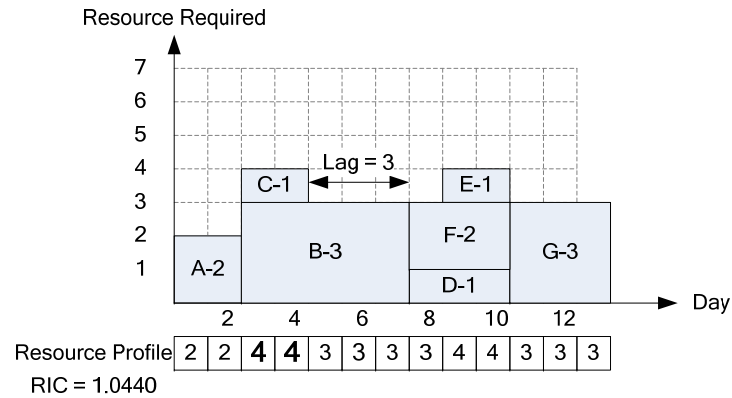
Sequence Step 2 Round 1. Calculate IF of Act C. **Don't forget to update the resource profile after shifting E!!**

FORWARD PASS: Showing the calculation of X and W for Act

C in SQS 2 Round 1

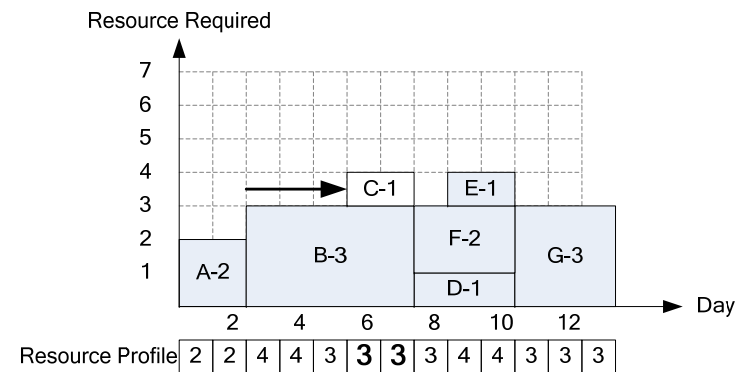
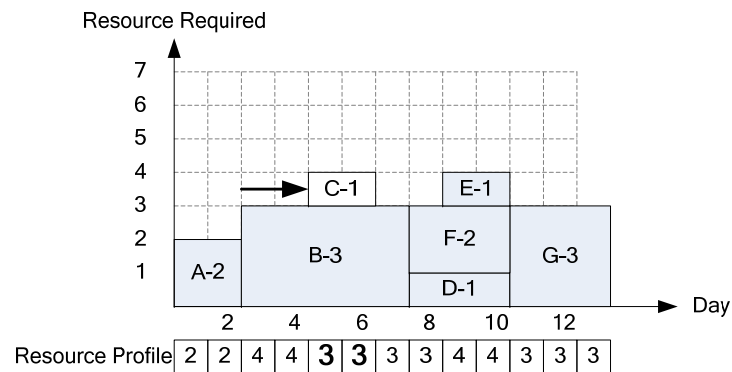
X = Sum of Resources in the range of
Act C **BEFORE** shifting = 4+4 = 8

W of 1 shifting day = Sum of Resources in the
range of Act C **AFTER** shifting by 1 day = 4+3 =
7



W of 2 shifting days = Sum of Resources in the
range of Act C **AFTER** shifting by 2 days = 3+3 =
6

W of 3 shifting days = Sum of Resources in the
range of Act C **AFTER** shifting by 3 days = 3+3 =
6



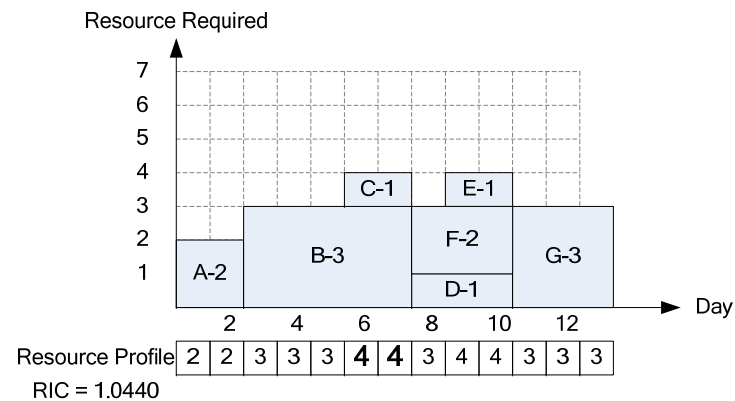
Act	S	r	X	W	m = min(S,T)	IF
-----	---	---	---	---	--------------	----

C	1	1	$4+4 = 8$	$4+3 = 7$	1	0
C	2	1	8	$3+3 = 6$	2	0
C	3	1	8	$3+3 = 6$	2	0

Thus, shift Act C by 3 days.

Sequence Step 1 Round 1. There is no non-critical activity in this SQS.

Final resource profile of FORWARD PASS



At this point, we have finished the forward pass. However, we have to repeat the process again in the opposite way, called BACKWARD PASS. The reason for this is to:

- 1) To check and improve resource profile
- 2) To check if activities can gain some floats back without increasing RIC (or fluctuation)

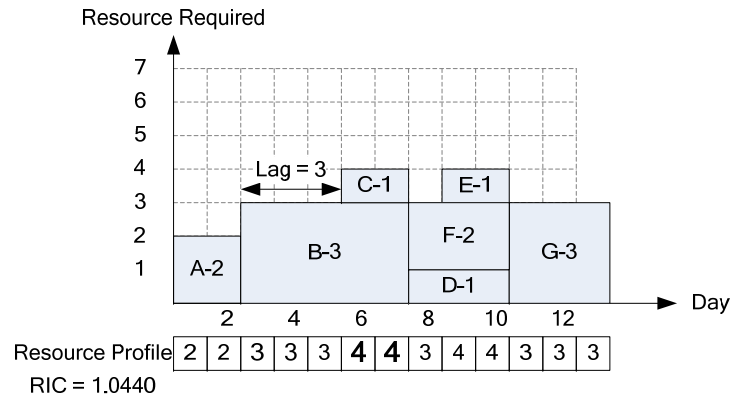
The process of BACKWARD PASS is exactly the same as FORWARD PASS, only this time the calculation start from the first sequence step to the last step, and activities are shifted from the right to left.

Backward Pass: Sequence Step 1. There is no non-critical activity.

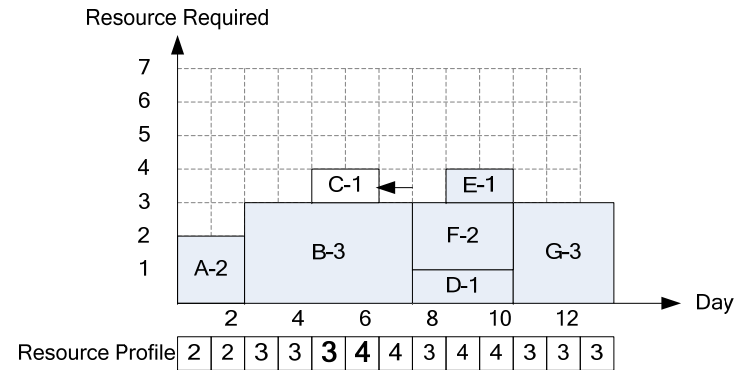
Backward Pass: Sequence Step 2 Round 1. Calculate IF of Act C.

BACKWARD PASS: Showing the calculation of X and W for Act C in SQS 2 Round 1

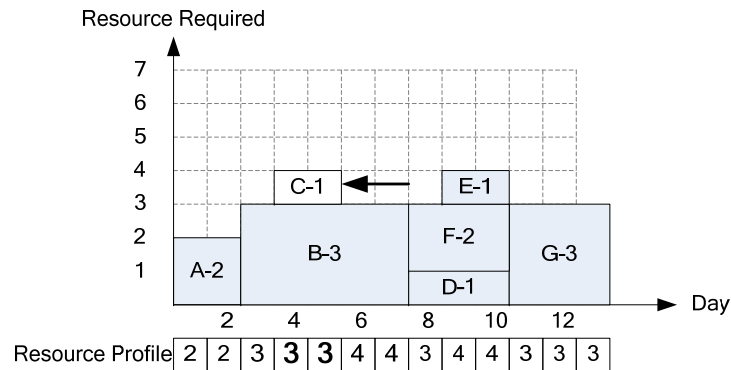
X = Sum of Resources in the range of
Act C BEFORE shifting = 4+4 = 8



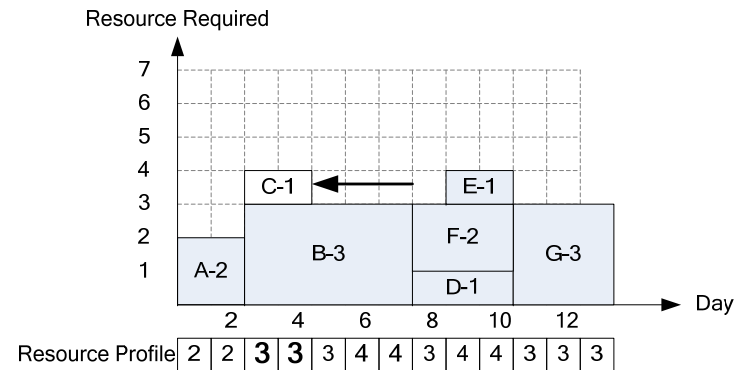
W of 1 shifting day = Sum of Resources in the
range of Act C AFTER shifting by 1 day = 4+3 =
7



W of 2 shifting days = Sum of Resources in the
range of Act C AFTER shifting by 2 days = 3+3 =
6



W of 3 shifting days = Sum of Resources in the
range of Act C AFTER shifting by 3 days = 3+3 =
6



Act	S	r	X	W	$m = \min(S,T)$	IF
C	1	1	$4+4 = 8$	$4+3 = 7$	1	0
C	2	1	8	$3+3 = 6$	2	0
C	3	1	8	$3+3 = 6$	2	0

Thus, shift Act C to the left by 3 days. New RIC = 1.0440.

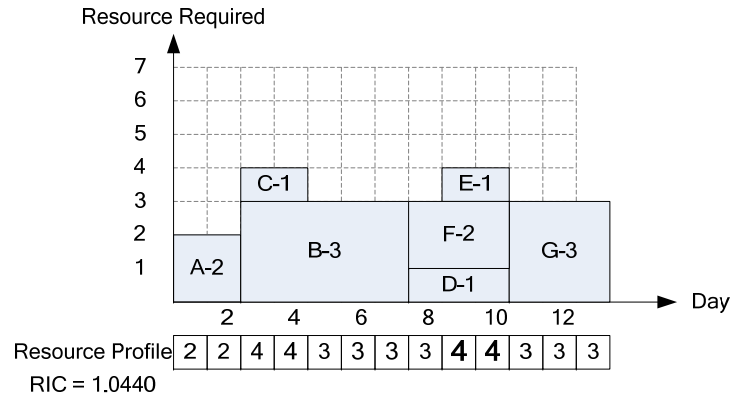
As you can see, RIC is the same. However, activity C gains some floats back.

Backward Pass: Sequence Step 3 Round 1.

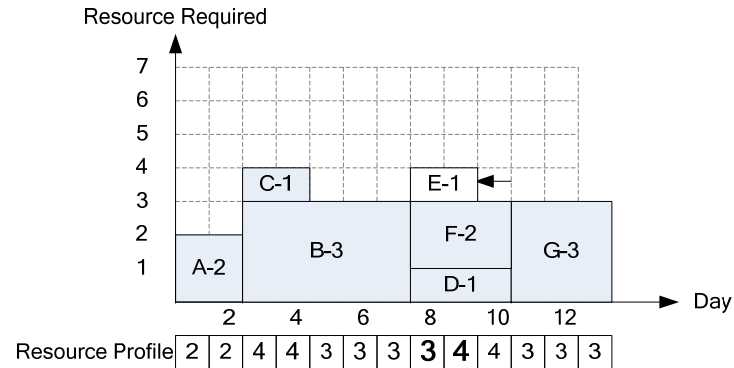
Calculate IF of Act E and F.

BACKWARD PASS: Showing the calculation of X and W for Act E in SQS 3 Round 1

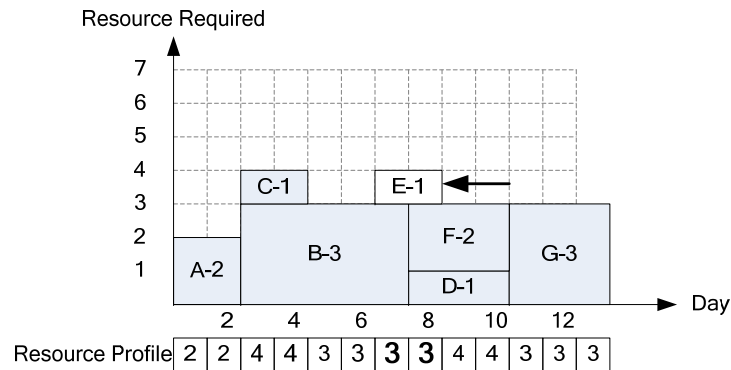
X = Sum of Resources in the range of
Act E BEFORE shifting = 4+4 = 8



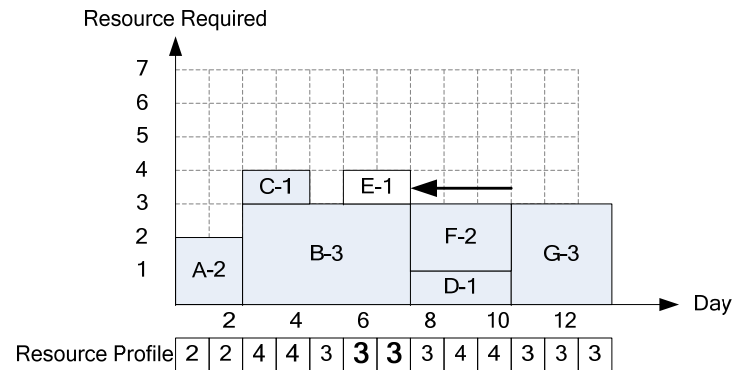
W of 1 shifting day = Sum of Resources in the
range of Act E AFTER shifting by 1 day = 3+4 =
7



W of 2 shifting days = Sum of Resources in the
range of Act E AFTER shifting by 2 days = 3+3 =
6



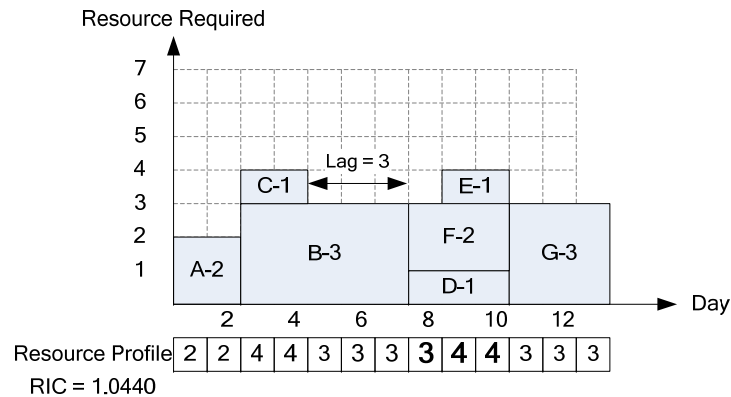
W of 3 shifting days = Sum of Resources in the
range of Act E AFTER shifting by 3 days = 3+3 =
6



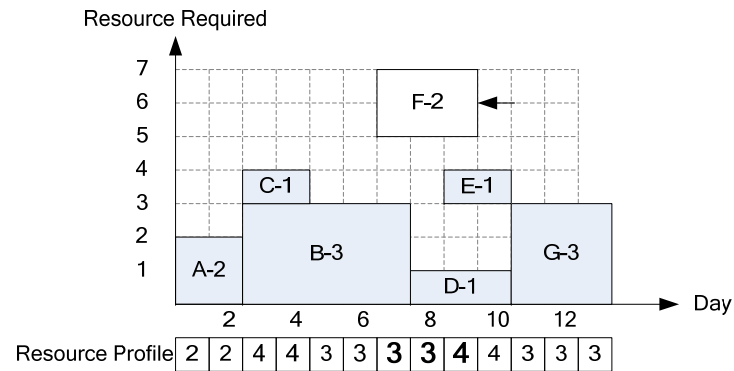
Note that IF of shifting Act E by 4 days is the same as shifting E by 3 days.

BACKWARD PASS: Showing the calculation of X and W for Act F in SQS 3 Round 1

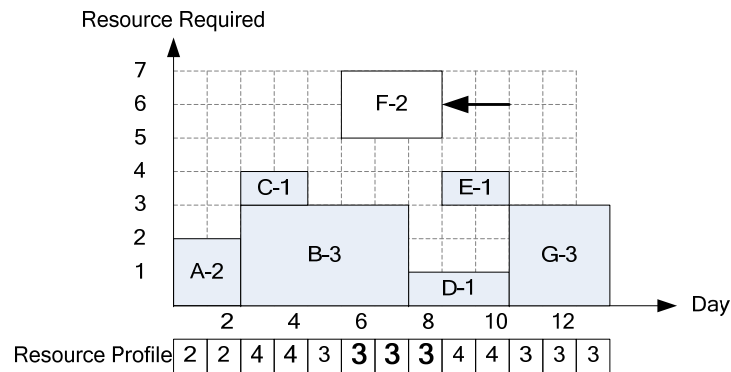
X = Sum of Resources in the range of
Act F **BEFORE** shifting = $3+4+4 = 11$



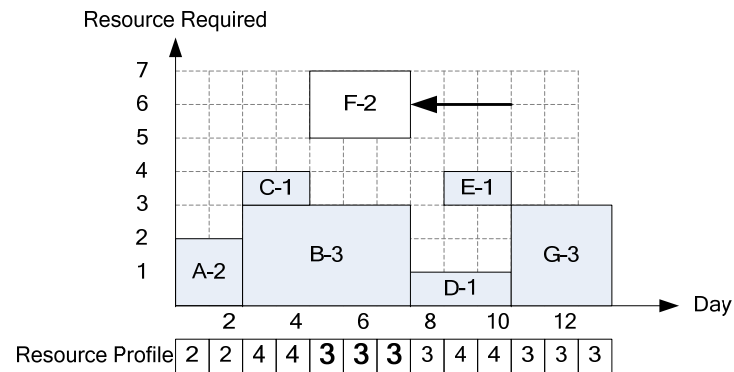
W of 1 shifting day = Sum of Resources in the
range of Act F **AFTER** shifting by 1 day = $3+3+4 = 10$



W of 2 shifting days = Sum of Resources in the
range of Act F **AFTER** shifting by 2 days = $3+3+3 = 9$



W of 3 shifting days = Sum of Resources in the
range of Act F **AFTER** shifting by 3 days = $3+3+3 = 9$



Act	S	r	X	W	$m = \min(S,T)$	IF
E	1	1	$4+4 = 8$	$4+3 = 7$	1	0
E	2	1	8	$3+3 = 6$	2	0
E	3	1	8	$3+3 = 6$	2	0
E	4	1	8	$3+3 = 6$	2	0
F	1	2	$4+4+3=10$	$4+3+3=10$	1	-4
F	2	2	10	$3+3+3=9$	2	-6
F	3	2	10	$1+3+3=9$	3	-10

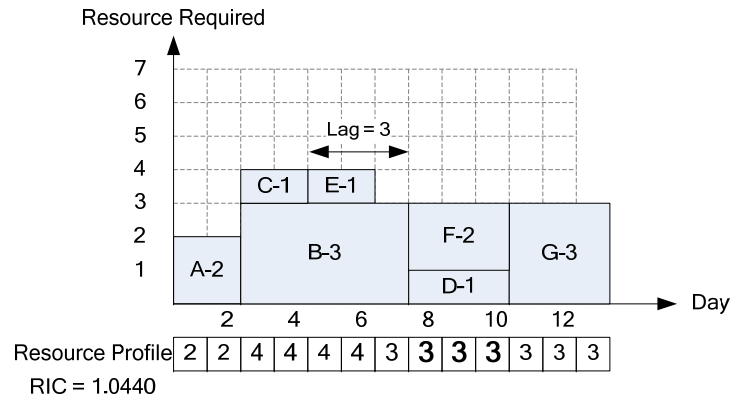
Thus, shift Act E backward by 4 days.

Backward Pass: Sequence Step 3 Round 2.

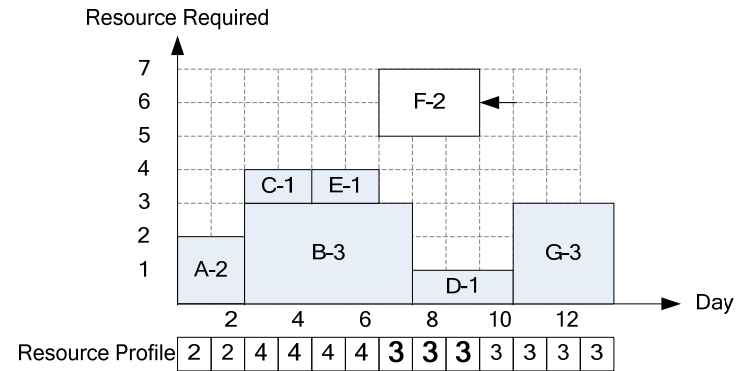
Calculate IF of Act F.

BACKWARD PASS: Showing the calculation of X and W for Act F in SQS 3 Round 2

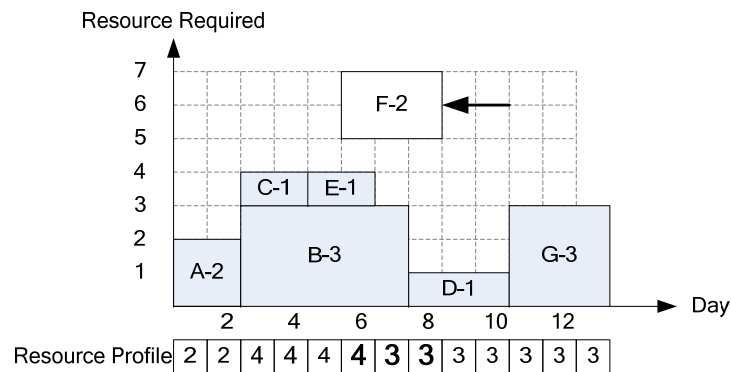
X = Sum of Resources in the range of
Act F BEFORE shifting = $3+3+3 = 9$



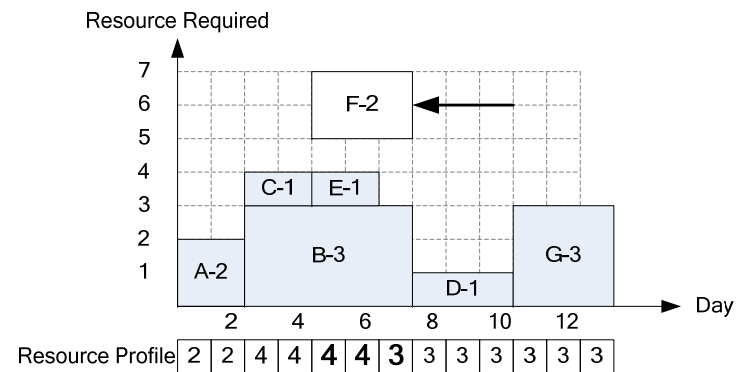
W of 1 shifting day = Sum of Resources in the
range of Act F AFTER shifting by 1 day = $3+3+3 = 9$



W of 2 shifting days = Sum of Resources in the
range of Act F AFTER shifting by 2 days = $4+3+3 = 10$



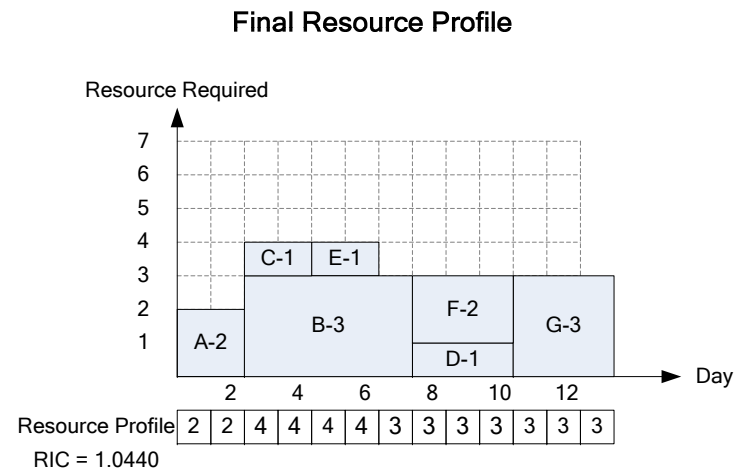
W of 3 shifting days = Sum of Resources in the
range of Act F AFTER shifting by 3 days = $4+4+3 = 11$



Act	S	r	X	W	$m = \min(S,T)$	IF
F	1	2	$3+3+3 = 9$	$3+3+3 = 9$	1	-4
F	2	2	9	$4+3+4 = 10$	2	-10
F	3	2	9	$4+4+3 = 11$	3	-16

Since all IF of Act F are negative, Act F stays in the same position. Move on to SQS 4.

Backward Pass: Sequence Step 4. Since there is no non-critical activity in this SQS, The calculation of MOM ends here at the backward pass of the last sequence step. YEAH!! Eventually, we have solved this problem of resource leveling using MOM.



Problem 9.1 Leveling the following network using Minimum Moment Method (MOM)

Activity	Predecessors	Duration	Resource Rate
A		6	5
B	A	2	7
C	A	11	2
D	A	3	4
E	B	5	4
F	B,C	4	5
G	B	2	9
H	E,F	5	3
I	D, F,G	4	6
J	C,G	3	5
K	H	4	0
L	H, I, J	4	3
M	K,L	7	7

Problem 9.2 Leveling the following network using Minimum Moment Method (MOM)

Activity	Predecessors	Duration	Resource Rate
A		2	7
B	A	3	2
C	A	5	2
D	B	2	3
E	B	4	1
F	C	3	2
G	C	10	2
H	E	4	5
I	E,F	2	3
J	D,H,I,G	3	6

Solution 9.1 Minimum Moment

Sequence Step 6. No activity can be shifted

Sequence Step 5. No activity can be shifted

SEQUENCE STEP 4 is very IMPORTANT please read carefully.

Sequence Step 4, there are 2 non-critical activities, I and J, that can be shifted. So calculate IF of activities I and J.

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37		
A	5	5	5	5	5	5																																	
B							7	7																															
C							2	2	2	2	2	2	2	2	2	2	2	2																					
D							4	4	4																														
E									4	4	4	4	4																										
F																	5	5	5	5																			
G								9	9																														
H																					3	3	3	3	3														
I																					6	6	6	6															
J																	5	5	5																				
K																												0	0	0	0								
L																											3	3	3	3									
M																																7	7	7	7	7	7	7	
Res	5	5	5	5	5	5	3	3	9	5	6	6	6	2	2	2	2	1	1	1	0	0	0	5	9	9	9	9	3	3	3	3	3	7	7	7	7	7	7

RIC = 1.3126

ACT	S	r	X	W	m	IF
I	1	6	36	30	1	0
J	1	5	30	25	1	0
J	2	5	30	24	2	-20
J	3	5	30	23	3	-40
J	4	5	30	27	3	-60
J	5	5	30	27	3	-60
J	6	5	30	21	3	-30

Activity with the highest Improvement Factor will be shifted. In this sequence step 4 round 1, both IF(I,1) and IF(J1) have the same IF of 0. However, we only shift one activity at a time.

So we choose the activity with a **higher resource rate**. Thus, shift activity I by one day.
 ??? What If they both have the SAME resource rate.??? Please check your course pack.

Sequence Step 4 the 2nd round. Since shifting activity I affects resource profiles within the possible range of activity J, which can be shifted; day 18 to day 26. As shown, resource profiles on day 22 and 26 have changed due to shifting activity I. Thue, we have to recalculate IF of activity J.

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
A	5	5	5	5	5	5																																
B							7	7																														
C							2	2	2	2	2	2	2	2	2	2	2																					
D							4	4	4																													
E									4	4	4	4	4																									
F																		5	5	5	5																	
G									9	9																												
H																						3	3	3	3	3												
I																							6	6	6	6												
J																		5	5	5																		
K																											0	0	0	0								
L																											3	3	3	3								
M																															7	7	7	7	7	7	7	
RES	5	5	5	5	5	5	13	13	19	15	6	6	6	2	2	2	2	10	10	10	5	3	9	9	9	9	3	3	3	3	7	7	7	7	7	7	7	

RIC = 1.3126

ACT	S	r	X	W	m	IF
J	1	5	30	25	1	0
J	2	5	30	18	2	10
J	3	5	30	17	3	-10
J	4	5	30	21	3	-30
J	5	5	30	27	3	-60
J	6	5	30	27	3	-60

Shift activity J by two days.

We do not have to check activity I again because it cannot be shifted any further from its current position.

Sequence step 3. Possibly shifted activities in Sequence Step 3 are E and G.

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
A	5	5	5	5	5	5																																
B							7	7																														
C							2	2	2	2	2	2	2	2	2	2	2																					
D							4	4	4																													
E									4	4	4	4	4																									
F																		5	5	5	5																	
G									9	9																												
H																						3	3	3	3	3												
I																							6	6	6	6												
J																				5	5	5																
K																											0	0	0	0								
L																											3	3	3	3								
M																															7	7	7	7	7	7	7	
RES	5	5	5	5	5	5	13	13	19	15	6	6	6	2	2	2	2	5	5	10	10	8	9	9	9	9	3	3	3	3	7	7	7	7	7	7	7	

RIC = 1.3009

ACT	S	r	X	W	m	IF
E	1	4	52	35	1	52
E	2	4	52	22	2	88
E	3	4	52	18	3	88
E	4	4	52	14	4	88
E	5	4	52	13	5	76
E	6	4	52	16	5	64
E	7	4	52	24	5	32
E	8	4	52	32	5	0

ACT	S	r	X	W	m	IF
G	1	9	34	21	1	36
G	2	9	34	12	2	36
G	3	9	34	12	2	36
G	4	9	34	8	2	72
G	5	9	34	4	2	108
G	6	9	34	4	2	108
G	7	9	34	4	2	108
G	8	9	34	7	2	81
G	9	9	34	10	2	54

For activity G, there are three shift positions having IF of 108. Shift to the further most in order to give room to its direct and indirect predecessors in earlier sequence step. Thus, shift activity G by 7 days.

Sequence Step 3 the 2nd round. Since shifting activity G affects resource profile within possible range, activity E can be shifted, so we have to recalculate IF of activity E.

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
A	5	5	5	5	5	5																																
B							7	7																														
C							2	2	2	2	2	2	2	2	2	2	2																					
D							4	4	4																													
E									4	4	4	4	4																									
F																		5	5	5	5																	
G																9	9																					
H																						3	3	3	3	3												
I																							6	6	6	6												
J																				5	5	5																
K																											0	0	0	0								
L																											3	3	3	3								
M																															7	7	7	7	7	7	7	
RES	5	5	5	5	5	5	13	13	10	6	6	6	6	2	2	11	11	5	5	10	10	8	9	9	9	9	3	3	3	3	7	7	7	7	7	7	7	

RIC = 1.174

We do not recalculate IF for activity G this time because it is the recent activity which is shifted. However, if you calculate it, you will get the same IF for those days having left for possible shifting position of the activity.

ACT	S	r	X	W	m	IF
E	1	4	34	26	1	16
E	2	4	34	22	2	16
E	3	4	34	27	3	-20
E	4	4	34	32	4	-56
E	5	4	34	31	5	-68
E	6	4	34	34	5	-80
E	7	4	34	42	5	-112
E	8	4	34	41	5	-108

Shift activity E by two days. Since shifting activity E by two days does not affect resource profile within possible range that activity G can be shifted, we do not have to recalculate IF of activity G again.

Assume that if activity E is shifted by three days, resource profile within range of possible shift of activity G will be affected, so we must recalculate IF of activity G. And not recalculate IF of activity E for now since we have just shifted it.

Sequence step 2. There are two non-critical activities, B and D, which can be shifted.

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
A	5	5	5	5	5	5																															
B							7	7																													
C							2	2	2	2	2	2	2	2	2	2	2																				
D							4	4	4																												
E											4	4	4	4	4																						
F																		5	5	5	5																
G																9	9																				
H																						3	3	3	3	3											
I																							6	6	6	6											
J																				5	5	5															
K																											0	0	0	0							
L																											3	3	3	3							
M																															7	7	7	7	7	7	7
RES	5	5	5	5	5	5	13	13	6	2	6	6	6	6	6	11	11	5	5	10	10	8	9	9	9	9	3	3	3	3	7	7	7	7	7	7	7

RIC= 1.1552

ACT	S	r	X	W	m	IF
B	1	7	26	19	1	0
B	2	7	26	8	2	28
D	1	4	32	21	1	28
D	2	4	32	14	2	40
D	3	4	32	14	3	24
D	4	4	32	18	3	8
D	5	4	32	18	3	8
D	6	4	32	18	3	8
D	7	4	32	23	3	-12
D	8	4	32	28	3	-32
D	9	4	32	27	3	-28
D	10	4	32	21	3	-4
D	11	4	32	20	3	0
D	12	4	32	25	3	-20
D	13	4	32	28	3	-32

Shift activity D by two days.

Sequence Step 2 the second time. Since shifting activity D affects resource profile within possible range in which activity B can be shifted, we have to recalculate IF of activity B.

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
A	5	5	5	5	5	5																																
B							7	7																														
C							2	2	2	2	2	2	2	2	2	2	2																					
D									4	4	4																											
E											4	4	4	4	4																							
F																		5	5	5	5																	
G																9	9																					
H																						3	3	3	3	3												
I																							6	6	6	6												
J																				5	5	5																
K																											0	0	0	0								
L																											3	3	3	3								
M																															7	7	7	7	7	7	7	
RES	5	5	5	5	5	5	9	9	6	6	10	6	6	6	6	11	11	5	5	10	10	8	9	9	9	9	3	3	3	3	7	7	7	7	7	7	7	

RIC= 1.1082

ACT	S	r	X	W	m	IF
B	1	7	18	15	1	-28
B	2	7	18	12	2	-56

There is no better position for B than the current position.
Since we do not shift activity B, resource profile remains the same.

Sequence step 1. There is no activity that can be shifted forward. —→

BACKWARD PASS

Sequence step 1. There is no activity that can be shifted backward. ←

Sequence step 2. Activity D. Since activity B cannot be shifted back (positioned any earlier than its current position), we do not have to calculate IF of activity B.

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
A	5	5	5	5	5	5																																
B							7	7																														
C							2	2	2	2	2	2	2	2	2	2	2																					
D									4	4	4																											
E											4	4	4	4	4																							
F																		5	5	5	5																	
G																9	9																					
H																						3	3	3	3	3												
I																							6	6	6	6												
J																				5	5	5																
K																											0	0	0	0								
L																											3	3	3	3								
M																															7	7	7	7	7	7	7	
Res	5	5	5	5	5	5	9	9	6	6	10	6	6	6	6	11	11	5	5	10	10	8	9	9	9	9	3	3	3	3	7	7	7	7	7	7	7	

RIC= 1.1082

ACT	S	r	X	W	m	IF
D	1	4	22	21	1	-12
D	2	4	22	24	2	-40

No shifting

Sequence step 3.
Activities E and G.

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
A	5	5	5	5	5	5																																
B							7	7																														
C							2	2	2	2	2	2	2	2	2	2	2																					
D									4	4	4																											
E											4	4	4	4	4																							
F																		5	5	5	5																	
G																9	9																					
H																						3	3	3	3	3												
I																							6	6	6	6												
J																				5	5	5																
K																											0	0	0	0								
L																											3	3	3	3								
M																																7	7	7	7	7	7	
Res	5	5	5	5	5	5	9	9	6	6	10	6	6	6	6	11	11	5	5	10	10	8	9	9	9	9	3	3	3	3	7	7	7	7	7	7	7	

RIC= 1.1082

ACT	S	r	X	W	m	IF
E	1	4	34	34	1	-16
E	2	4	34	34	2	-32
G	1	9	22	17	1	-36
G	2	9	22	12	2	-72
G	3	9	22	12	2	-72
G	4	9	22	12	2	-72
G	5	9	22	16	2	-108
G	6	9	22	16	2	-108
G	7	9	22	12	2	-72

No shifting

Sequence step 4. Activities E and G.

DAY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
A	5	5	5	5	5	5																															
B							7	7																													
C							2	2	2	2	2	2	2	2	2	2	2																				
D									4	4	4																										
E											4	4	4	4	4																						
F																		5	5	5	5																
G																9	9																				
H																						3	3	3	3	3											
I																							6	6	6	6											
J																				5	5	5															
K																											0	0	0	0							
L																											3	3	3	3							
M																															7	7	7	7	7	7	7
Res	5	5	5	5	5	5	9	9	6	6	10	6	6	6	6	11	11	5	5	10	10	8	9	9	9	9	3	3	3	3	7	7	7	7	7	7	7

RIC= 1.1082

ACT	S	r	X	W	m	IF
I	1	6	36	35	1	-30
J	1	5	28	25	1	-10
J	2	5	28	20	2	-10

No shifting

RIC = 1.1082 (originally 1.3126)

Max Resource Requirement in day is 11 (originally 19)

Sequence step 5. There is no activity that could be possibly shifted backward.

Sequence step 6. There is no activity that could be possibly shifted backward.

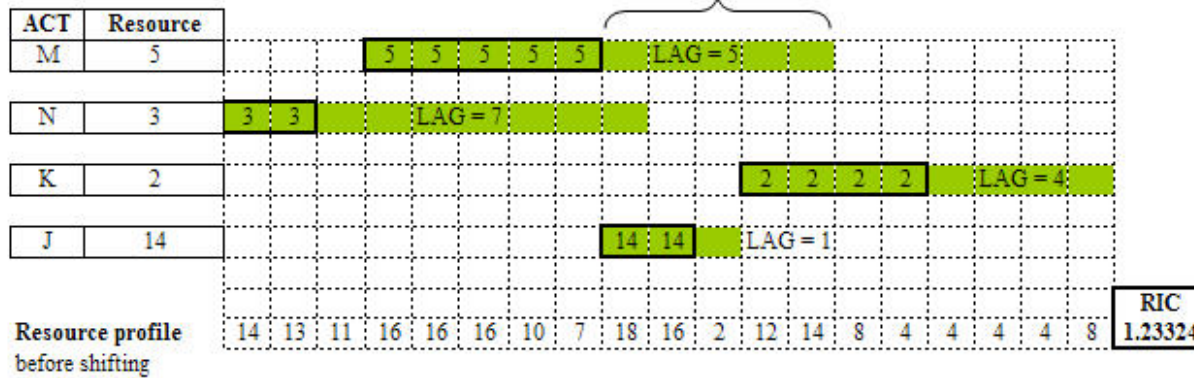
In this case since there is no shifting during backward pass, there is high chance that this is the best result for Minimum Moment Method algorithm. If there is shifting during backward pass, we should do forward and backward passes again. However, the result would not be much improved.

The following is a part of a network that is resource-leveled by using **Minimum Moment Method**. This example shows how to **save time** in calculating Improvement Factor in the same sequence step where many non-critical activities exist.

Use Minimum Moment Method to leveling this network.

Let activities M, N, K, and J are in the same sequence step, SQS 5

Range the Activity M could be shifted



SQS 5 ROUND 1

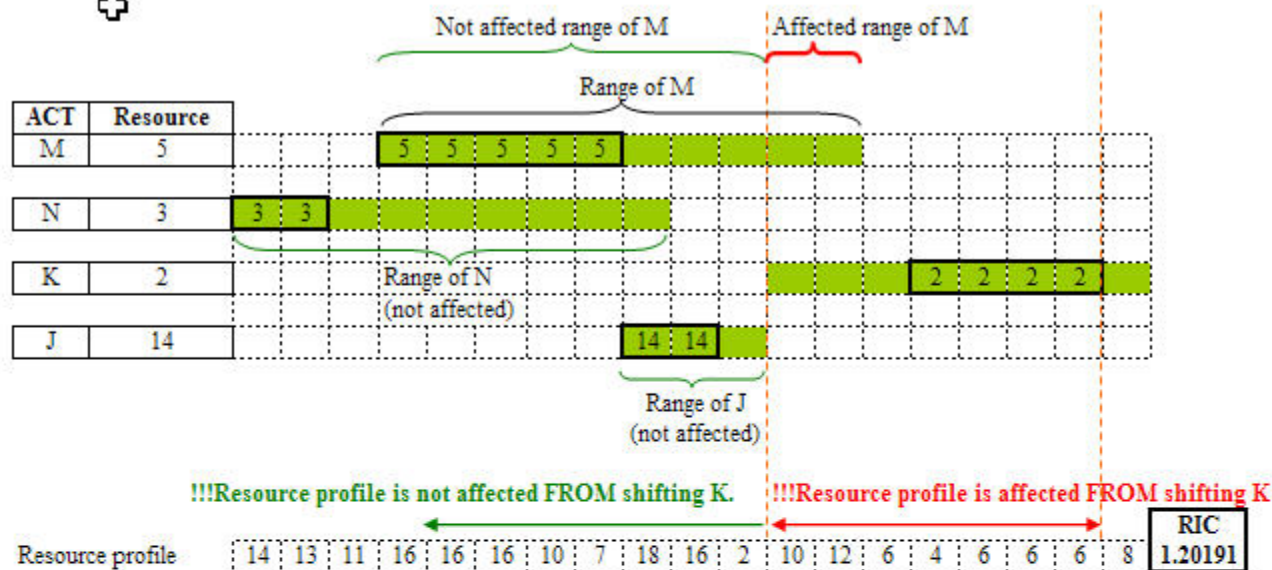
ACT	S	r	X	W	m	IF
M	1	5	65	67	1	-35
M	2	5	65	67	2	-60
M	3	5	65	53	3	-15
M	4	5	65	55	4	-50
M	5	5	65	62	5	-110

ACT	S	r	X	W	m	IF
N	1	3	27	24	1	0
N	2	3	27	27	2	-18
N	3	3	27	32	2	-33
N	4	3	27	32	2	-33
N	5	3	27	26	2	-15
N	6	3	27	17	2	12
N	7	3	27	25	2	-12

ACT	S	r	X	W	m	IF
K	1	2	38	30	1	12
K	2	2	38	20	2	28
K	3	2	38	16	3	32
K	4	2	38	20	4	20

ACT	S	r	X	W	m	IF
J	1	14	34	18	1	28

SO SHIFTING ACTIVITY K BY 3 DAYS



SQS 5 ROUND 2

!!!Since activity K is just shifted IN THE LAST ROUND, so it is not necessary to to recalculate IF of K.

!!!There is NO NEED TO RECALCULATE Improvement Factor of N and J, because their RANGE are on the NOT AFFECTED side. You can just use their previously calculated IF.

ACT	S	r	X	W	m	IF
M	1	5	65	67	1	-35
M	2	5	65	67	2	-60
M	3	5	65	53	3	-15
M	4	5	65	53	4	-40
M	5	5	65	58	5	-90

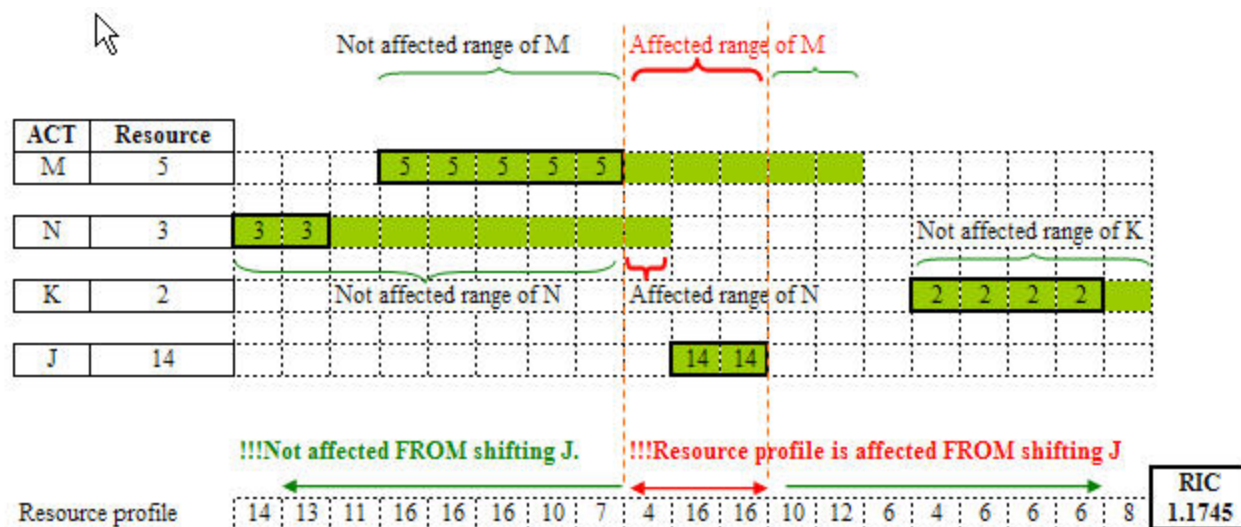
ACT	S	r	X	W	m	IF
N	1	3	27	24	1	0
N	2	3	27	27	2	-18
N	3	3	27	32	2	-33
N	4	3	27	32	2	-33
N	5	3	27	26	2	-15
N	6	3	27	17	2	12
N	7	3	27	25	2	-12

As explained above, if you recalculate IF of N, IF of N will be all the same.

SO SHIFTING ACTIVITY J BY 1 DAYS

ACT	S	r	X	W	m	IF
J	1	14	34	18	1	28

As explained above, if you recalculate IF of J, IF of J will be all the same.



SQS 5 ROUND 3

!!!Since activity J is just shifted IN THE LAST ROUND, so it is not necessary to to recalculate IF of J.
In this round we have to calculate Improvement Factor of M, N, and K.

ACT	S	r	X	W	m	IF
M	1	5	65	53	1	35
M	2	5	65	53	2	10
M	3	5	65	53	3	-15
M	4	5	65	53	4	-40
M	5	5	65	58	5	-90

DOES IT MAKE SENSE TO YOU

WHY Improvement Factor of shifting M by 1 day become positive?

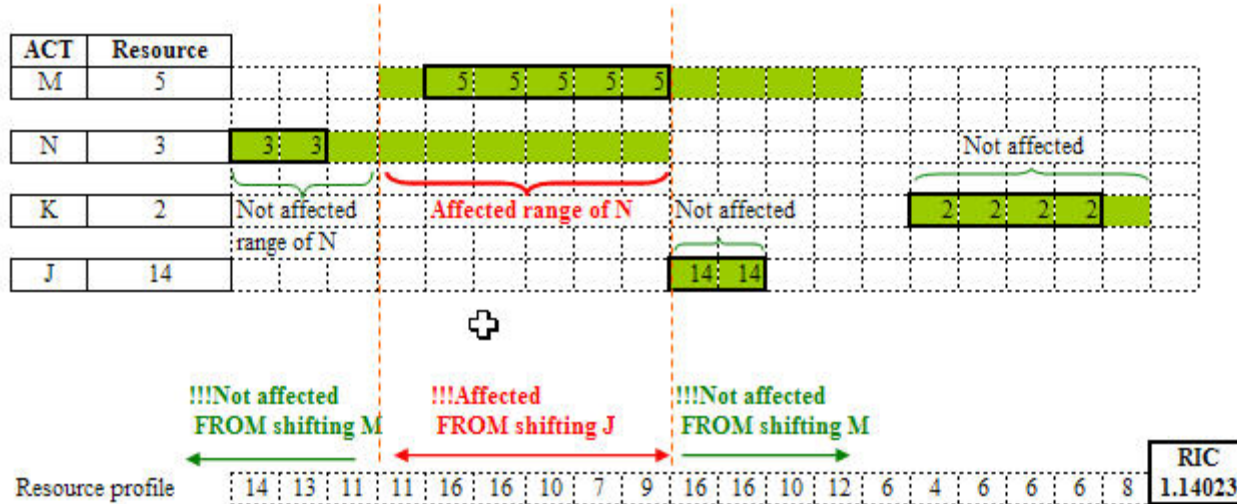
Affected range of M

ACT	S	r	X	W	m	IF
K	1	2	22	26	1	-12

ACT	S	r	X	W	m	IF
N	1	3	27	24	1	0
N	2	3	27	27	2	-18
N	3	3	27	32	2	-33
N	4	3	27	32	2	-33
N	5	3	27	26	2	-15
N	6	3	27	17	2	12
N	7	3	27	11	2	30

SO SHIFTING ACTIVITY M BY 1 DAY

Affected range of N



SQS 5 ROUND 4

!!!Since activity M is just shifted IN THE LAST ROUND, so it is not necessary to recalculate IF of M.

In this round we have to calculate Improvement Factor of N.

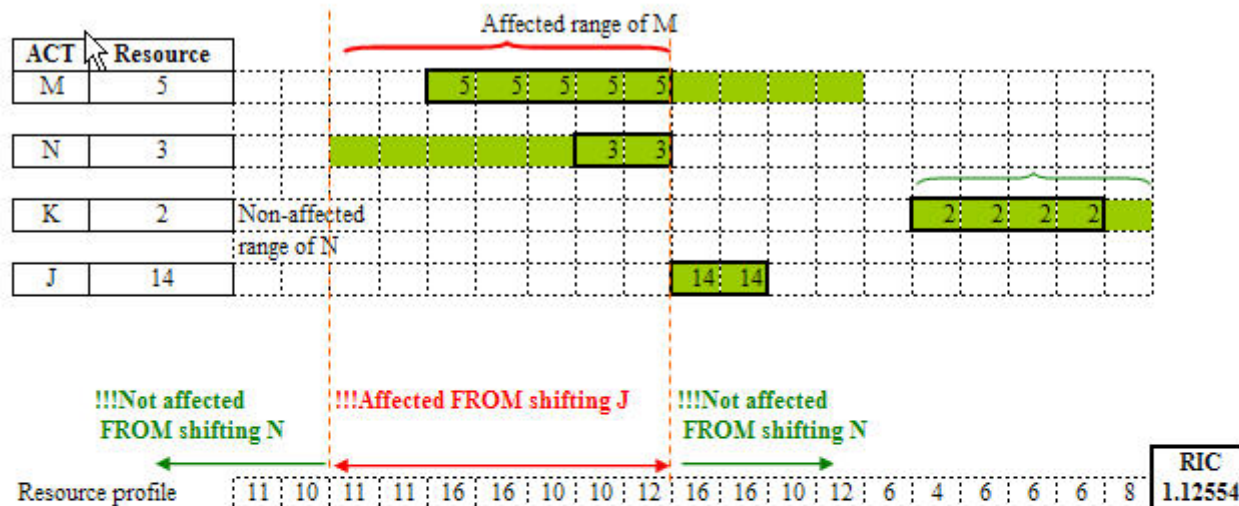
Since activity J cannot be shifted anymore, there is no need to consider shifting J.

ACT	S	r	X	W	m	IF
K	1	2	22	26	1	-12

(Copied from previous round)

ACT	S	r	X	W	m	IF
N	1	3	27	24	1	0
N	2	3	27	22	2	-3
N	3	3	27	27	2	-18
N	4	3	27	32	2	-33
N	5	3	27	26	2	-15
N	6	3	27	17	2	12
N	7	3	27	16	2	15

SO SHIFTING ACTIVITY N BY 7 DAYS



SQS 5 ROUND 5

!!!Since activity N is just shifted IN THE LAST ROUND, so it is not necessary to to recalculate IF of N.

In this round we have to calculate Improvement Factor of M, and K.

Since activity J cannot be shifted anymore, there is no need to consider shifting J.

Because resource profile in the range of activity K is not affected from shifting N, IF of K is still the same.

ACT	S	r	X	W	m	IF
M	1	2	64	64	1	-4
M	2	5	64	64	2	-50
M	3	5	64	64	3	-75
M	4	5	64	66	4	-110

ACT	S	r	X	W	m	IF
K	1	2	22	26	1	-12

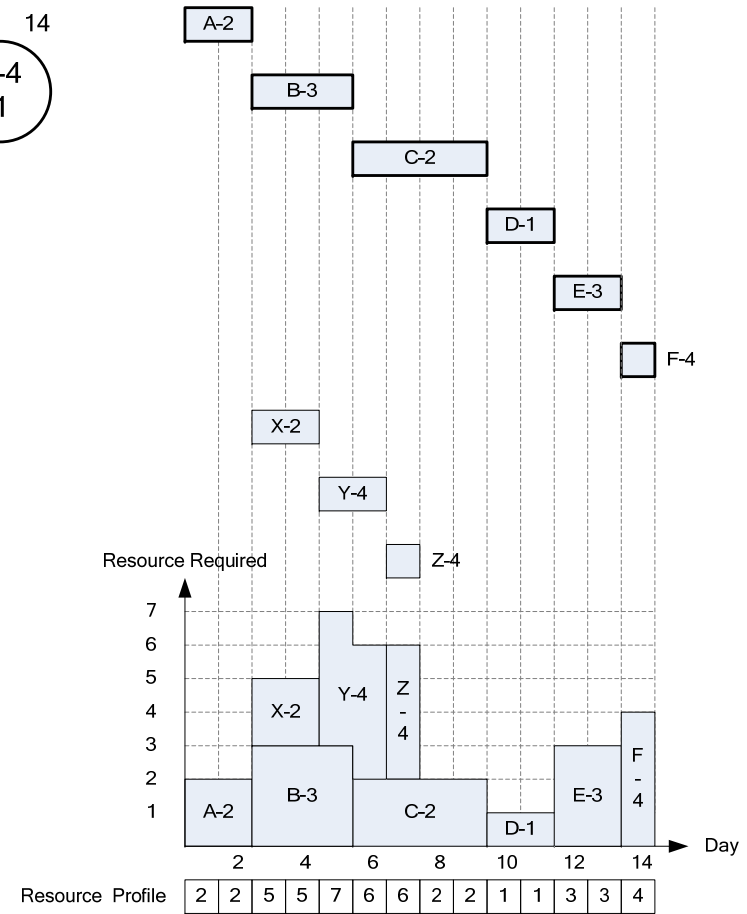
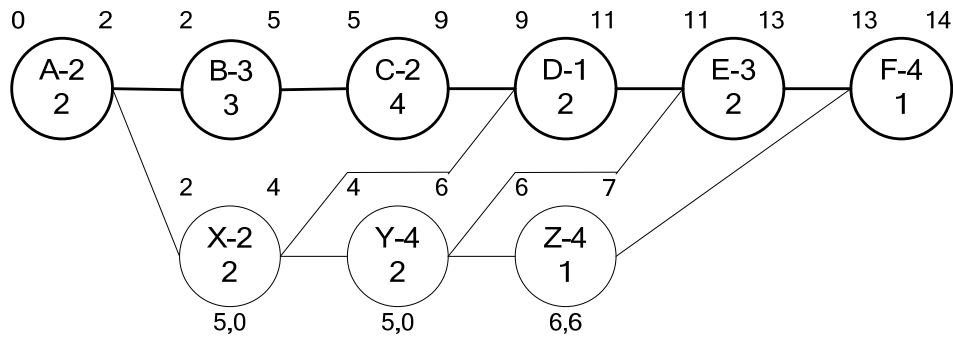
Can you guess why IF of shifting K by 1 day is NEGATIVE?

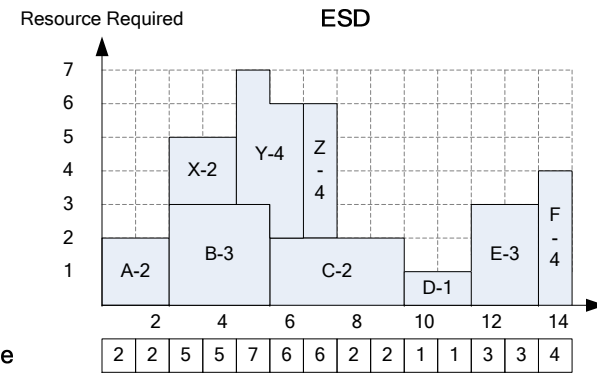
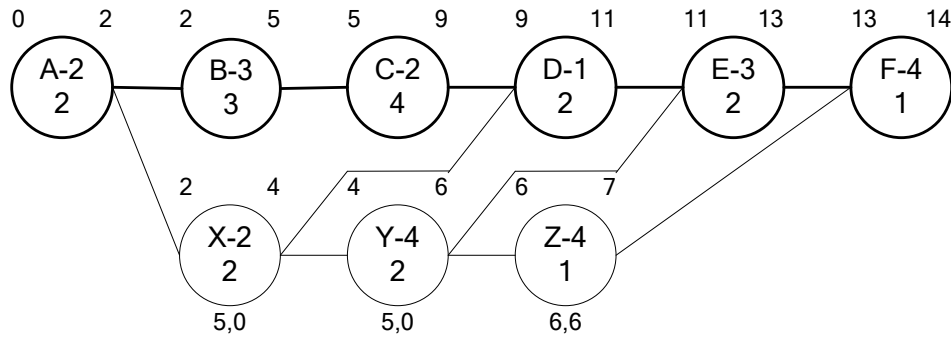
NO SHIFTING ----> FINISH LEVELING in this SQS. YEAH!!!! FINALLY.

10.RESOUCCE LEVELING

- PACK

Example of PACK (an unlimited resource leveling method)

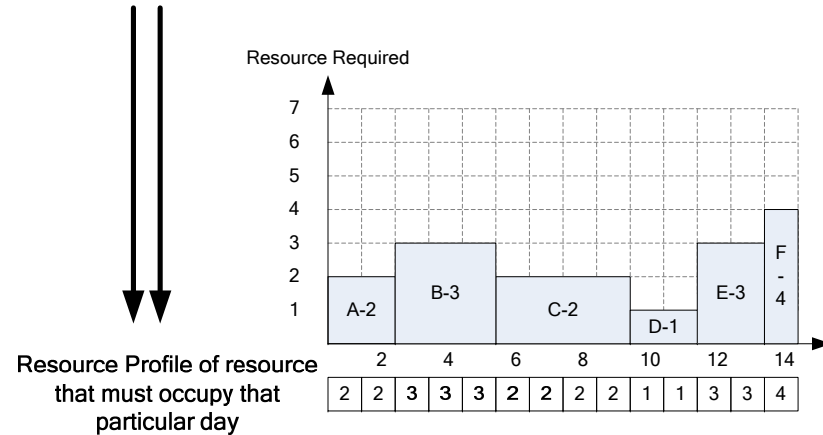




PRIORITIZATION RULES FOR PACK METHOD

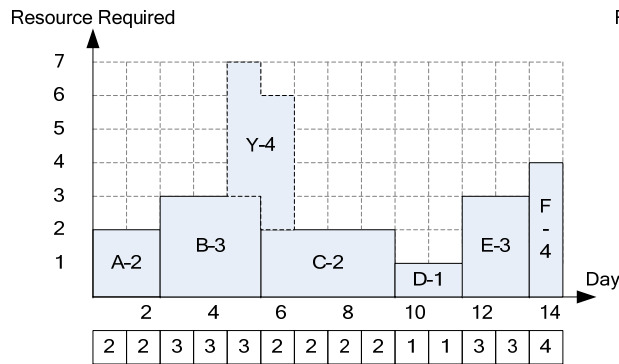
- Resource required/day from Max to Min
- Total float from Min to Max
- Sequence step from Max to Min

PROCESSING QUEUE				
NO	ACT	RES	TF	SQS
1	Y	4	5	3
2	Z	4	6	4
3	X	2	5	2

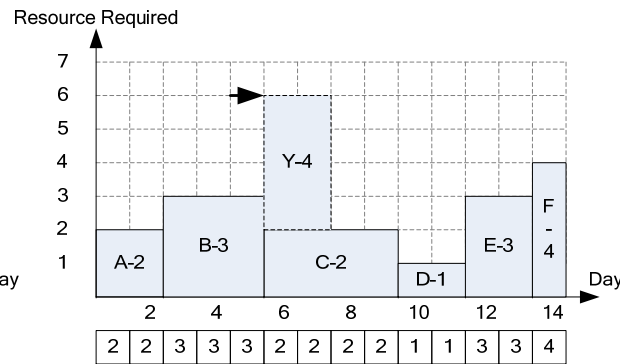


In the beginning, resource profile of PACK only includes resources that must occupy that particular day. Thus, most of resources in this resource profile are of critical activities, and some of non-critical activities whose duration is shorter than TF.

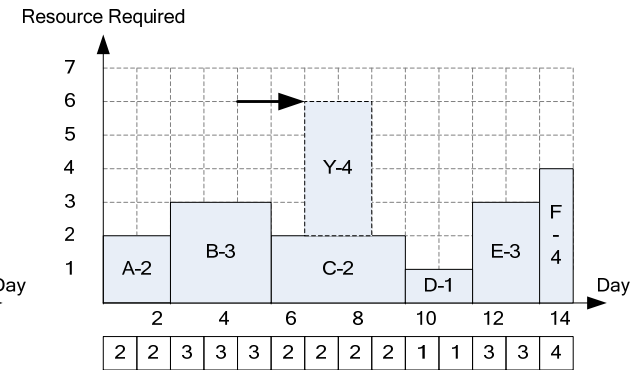
Tentative position of Y without shifting



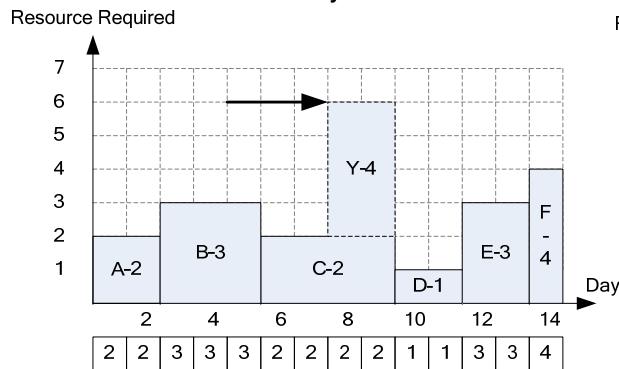
Tentative position of Y with 1 shifting day



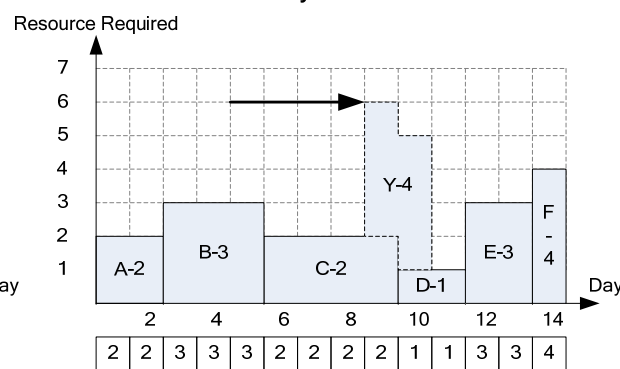
Tentative position of Y with 2 shifting days



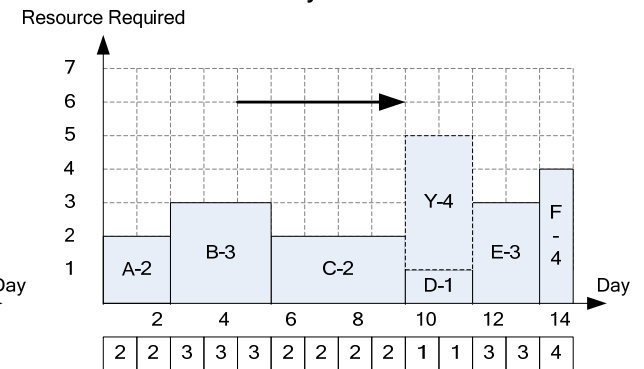
Tentative position of Y with 3 shifting days



Tentative position of Y with 4 shifting days



Tentative position of Y with 5 shifting days



Question: From the above figures, why do we prefer position 10-11 for act Y ??? Answer: it provides the minimum sum of resource (1+1=2). J

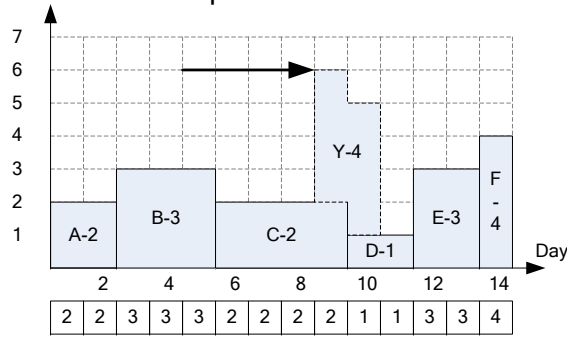
Note that: including resource of Y into the profile does not change our answer. WHY?

Can we improve this very simple rule a little bit further while it is still possible for manual calculation?

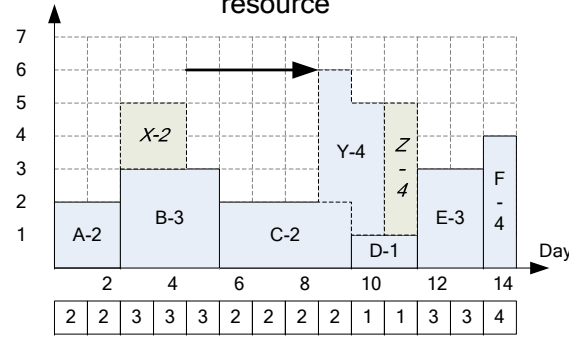
YES!! Let's consider PRED and SUCC of Y while trying to schedule activity Y. (These PRED and SUCC are non-critical activities)

Tentative position of Y
with 4 shifting days

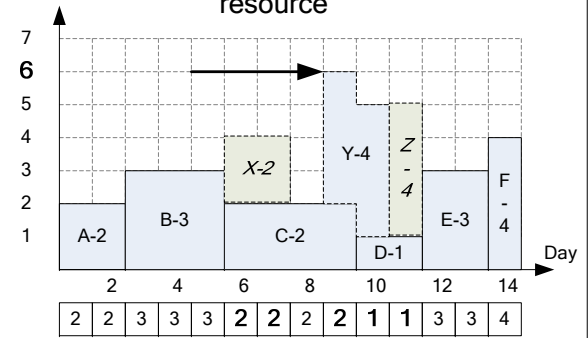
Without considering Y's PRED and SUCC positions



BEFORE placing Y's PRED and SUCC in a position with minimum sum of resource



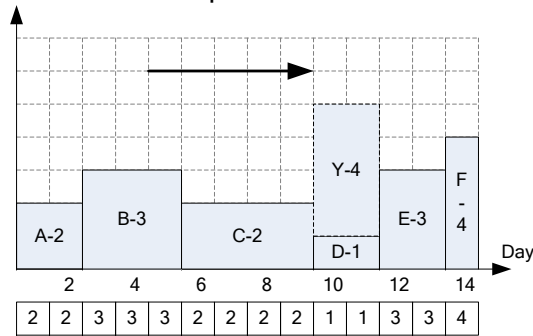
AFTER placing Y's PRED and SUCC in a position with minimum sum of resource



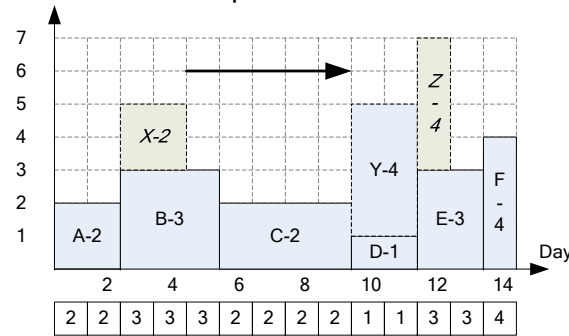
Sum of Res = 2+2+2+1+1 = 8

Tentative position of Y
with 5 shifting days

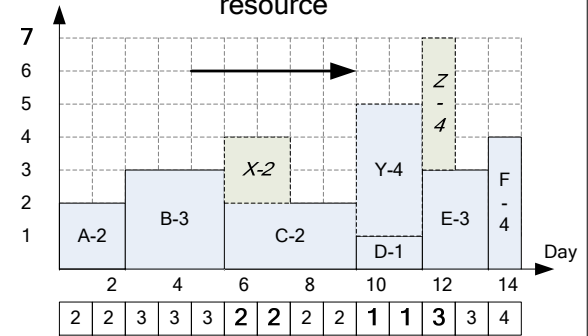
Without considering Y's PRED and SUCC positions



Without considering Y's PRED and SUCC positions



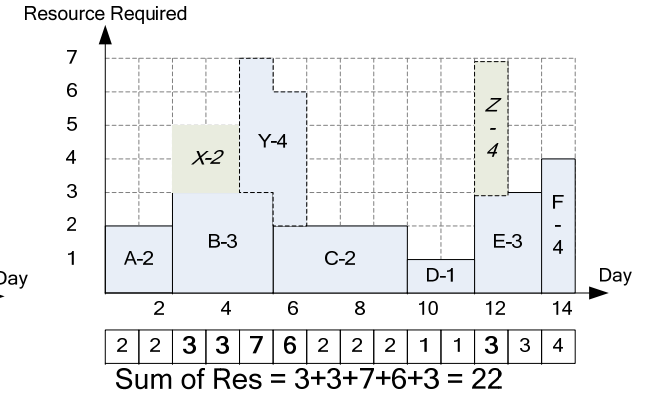
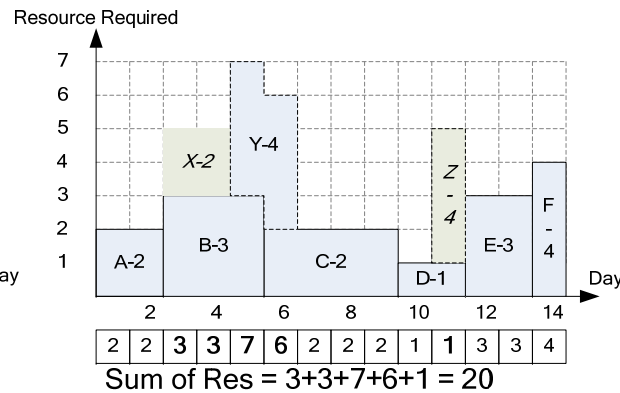
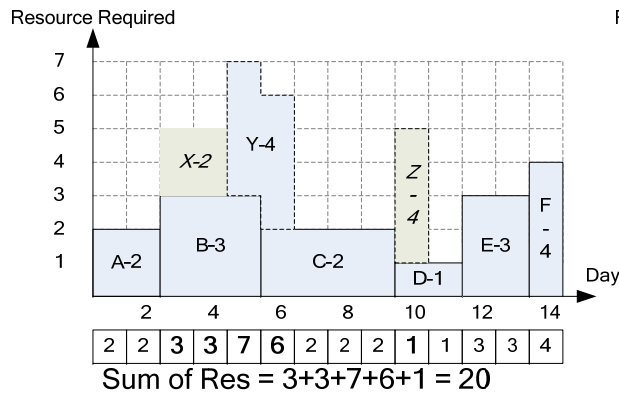
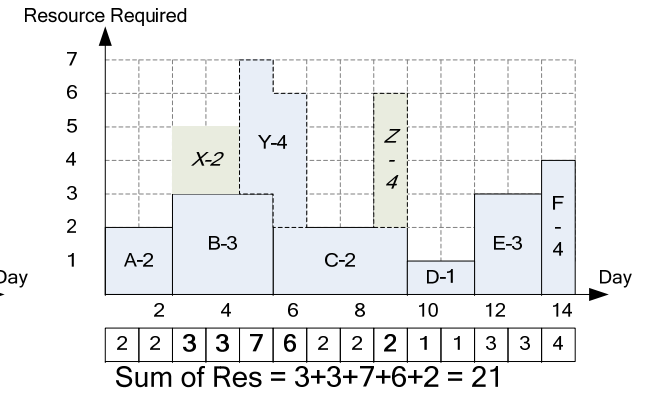
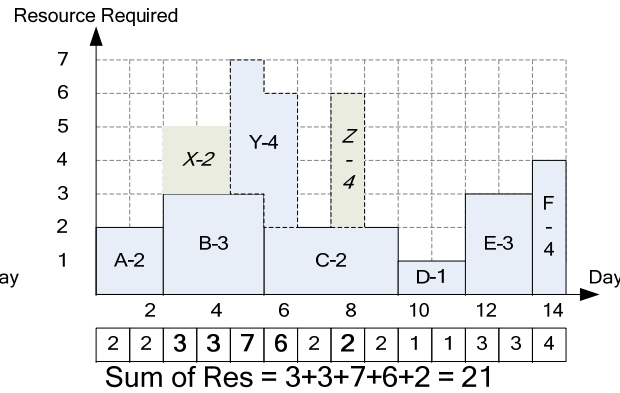
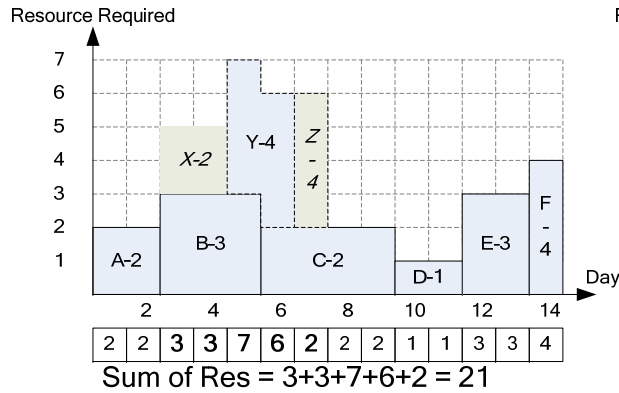
AFTER placing Y's PRED and SUCC in a position with minimum sum of resource

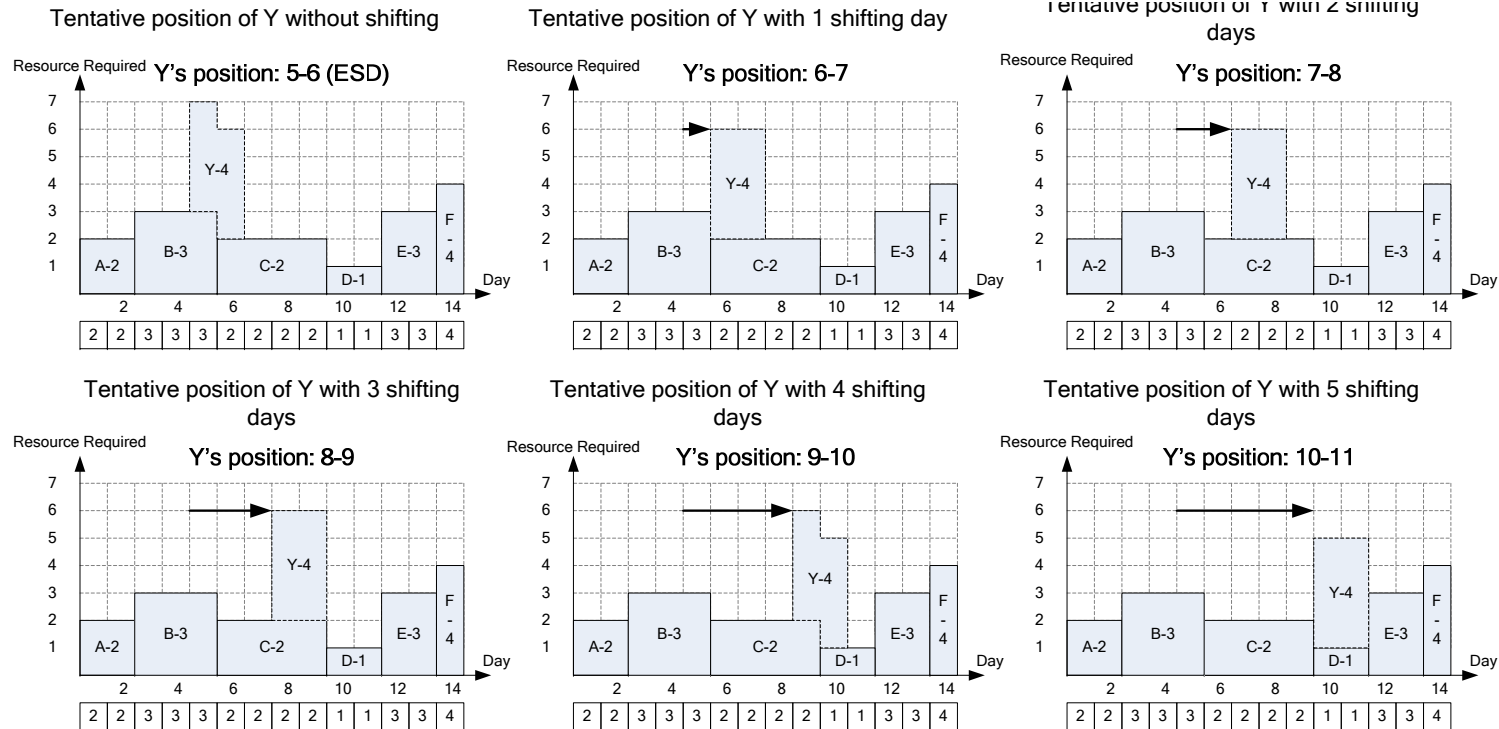


Sum of Res = 2+2+1+1+3 = 9

As shown, the best position of Y in position 10-11 is not the best position for the overall project after considering the possibility of scheduling its non-critical PRED and SUCC.

Example of finding a minimum sum of resources for Y in position 5-6.



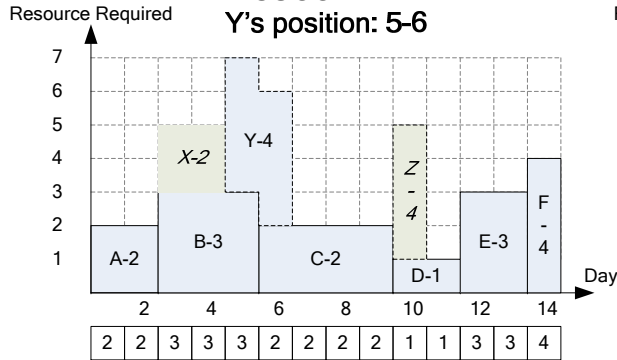


Y's position	$\sum RS(Y)$	X's best position	Min $\sum RS(X)$	Z's best position	Min $\sum RS(Z)$	$\sum$
5-6 (ESD)	3+2=5	3-4	3+3=6	10	1	12
6-7	2+2=4	3-4	6	10	1	9
7-8	2+2=4	4-5	3+2=5	10	1	10
8-9	2+2=4	5-6	2+2=4	10	1	9
9-10	2+1=3	5-6	4	11	1	8 ← Min
10-11	1+1=2	5-6	4	12	3	9

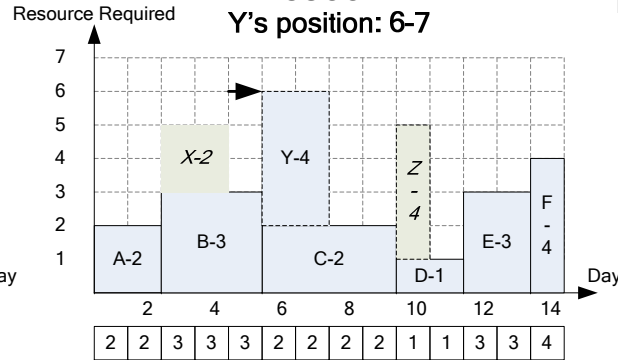
Thus, we will place activity Y on day 9-10. Then, do the same calculation for activity Z and X. However, the calculation becomes a lot easier and faster since they, both, have no non-critical PRED and SUCC that haven't been assigned. You can just look at your updated resource profile and assign both activities on those days with minimum resource.

Graphical explanation for the previous page.

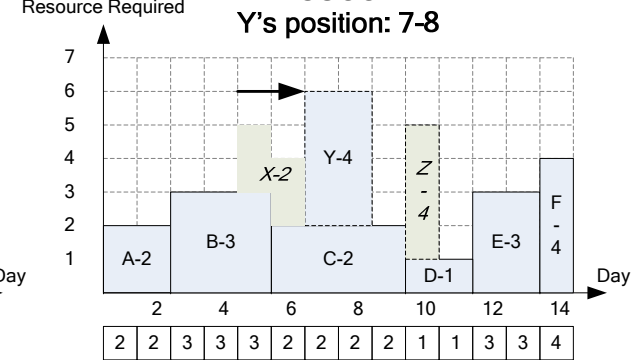
Tentative position of Y without shifting,
and best position for its PRED and
SUCC



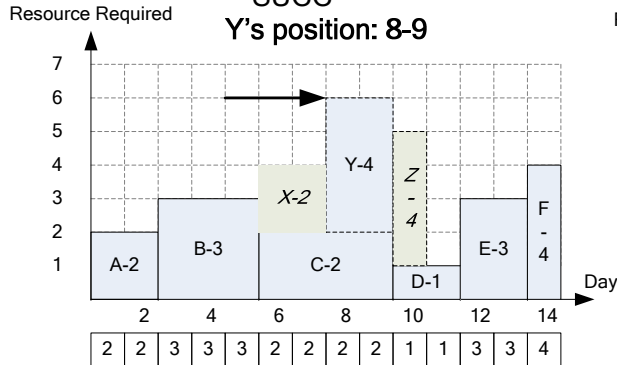
Tentative position of Y with 1 shifting day, and
best position for its non-critical PRED and
SUCC



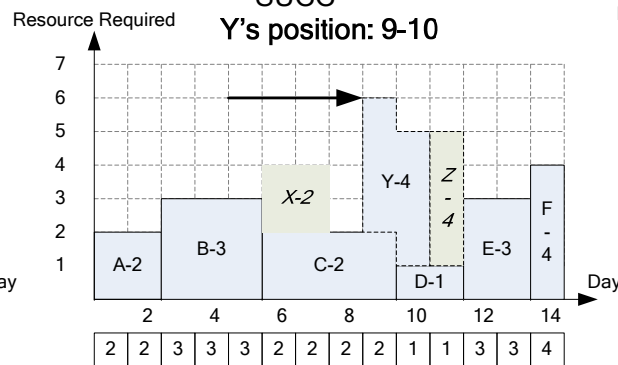
Tentative position of Y with 2 shifting days, and
best position for its non-critical PRED and
SUCC



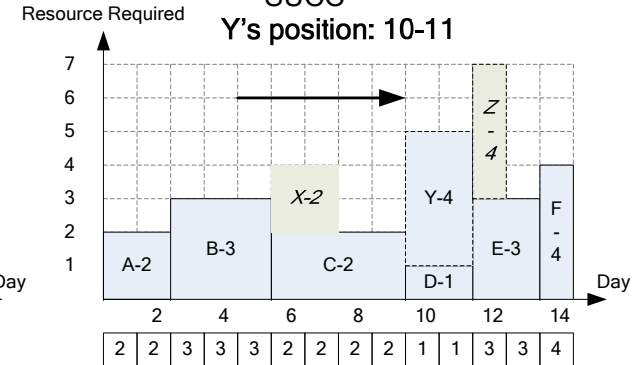
Tentative position of Y with 3 shifting days, and
best position for its non-critical PRED and
SUCC



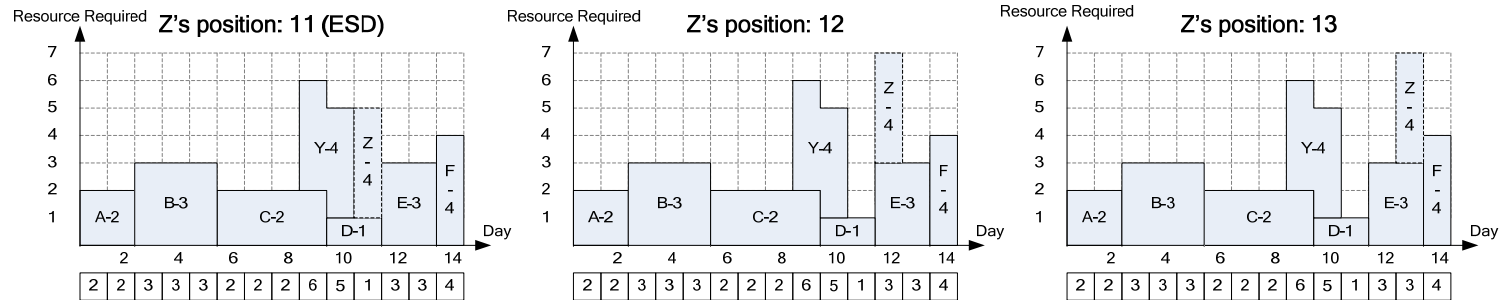
Tentative position of Y with 4 shifting days, and
best position for its non-critical PRED and
SUCC



Tentative position of Y with 5 shifting days, and
best position for its non-critical PRED and
SUCC



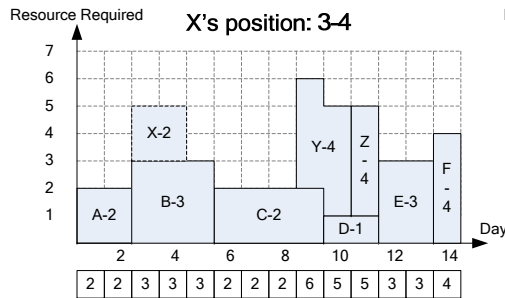
After schedule Y, the next non-critical activity that will be scheduled is Z according to PACK's prioritization rules (called Processing Queue)



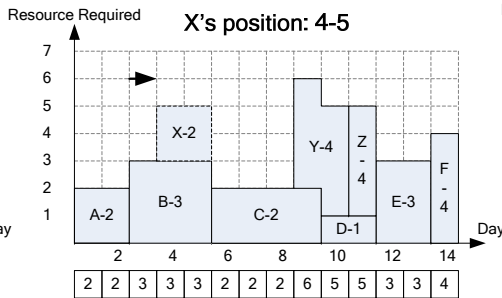
Z's position	$\sum RS(Z)$	$\sum$
11	1	$1 \leftarrow \text{Min}$
12	3	3
13	3	3

Next, schedule activity X.

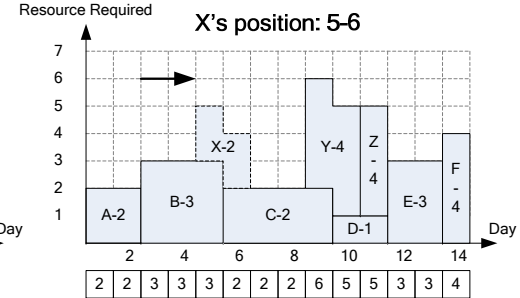
Tentative position of X without shifting



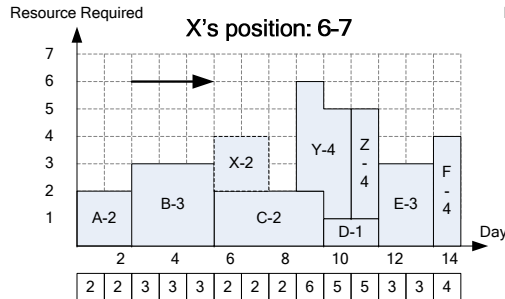
Tentative position of X with 1 shifting day



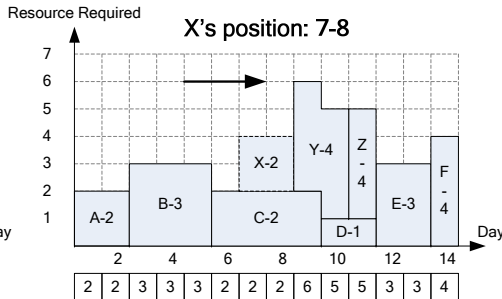
Tentative position of Y with 2 shifting days



Tentative position of X with 3 shifting days



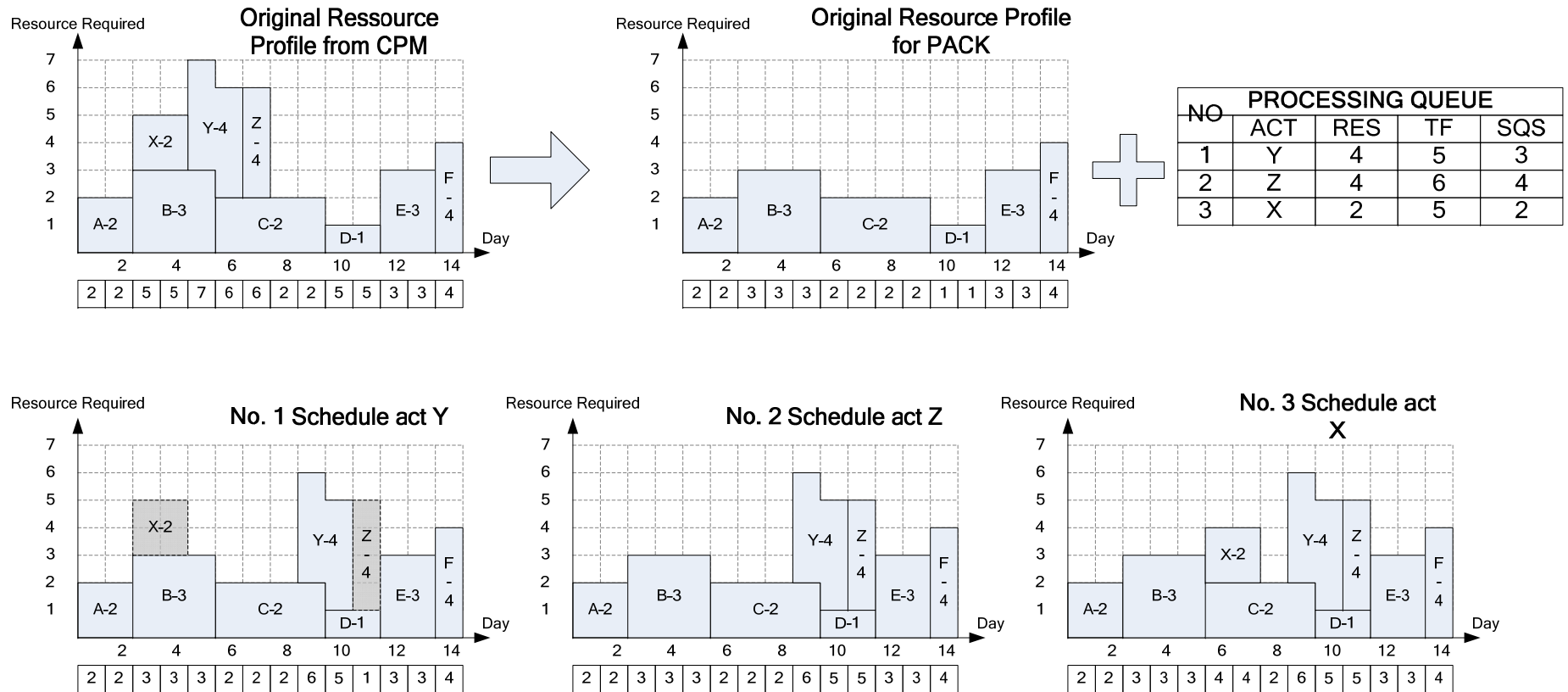
Tentative position of X with 4 shifting days



X's position	$\sum RS(X)$	$\sum$
3-4	$3+3=6$	6
4-5	$3+3=6$	6
5-6	$3+2=5$	5
6-7	$2+2=4$	$4 \leftarrow \text{Min} \leftarrow \text{Choose this one!!}$
7-8	$2+2=4$	$4 \leftarrow \text{Min}$

If there is a tie in sum of resource, we choose the earlier position because it gives the activity more float.

SUMMARY



PACK'S CALCULATION TABLE

SQS	RES	AC	JC	DUR	ACT	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1					A														
2					B														
3					C														
4					D														
5					E														
6					F														
2					X														
3					Y														
4					Z														
No			assign		RS														
1																			
2																			
3																			
RIC =																			

Problem 10.1 Leveling the following network using PACK

Activity	Predecessors	Duration	Resource Rate
A		6	5
B	A	2	7
C	A	11	2
D	A	3	4
E	B	5	4
F	B,C	4	5
G	B	2	9
H	E,F	5	3
I	D, F,G	4	6
J	C,G	3	5
K	H	4	0
L	H, I, J	4	3
M	K,L	7	7

Solution for 10.1 PACK problem. (Note: Because of the limited page space, day 1 to 7 and day 31 to 37 are not shown here.)

SQS	RES	AC	JC	DUR	ACT	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	5	0	0	6	A																								
2	2	0	0	11	C	1	1	1	1	1	1	1	1	1	1	1													
3	5	0	0	4	F												1	1	1	1									
4	3	0	0	5	H																1	1	1	1	1				
5	0	0	0	4	K																					1	1	1	1
5	3	0	0	4	L																					1	1	1	1
6	7	0	0	7	M																								
2	7	8		2	B	2	2	2	2	2	2	2	2	2	2														
2	2			13	D	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2									
3	4	8		5	E			2	2	2	2	2	2	2	2	2	2	2	2	2									
3	9	11		2	G			2	2	2	2	2	2	2	2	2	2	2	2	2	2								
4	6	1		4	I																2	1	1	1	2				
4	5	6		3	J												2	2	2	2	2	2	2	2	2				
No			assign		RS	2	2	2	2	2	2	2	2	2	2	2	5	5	5	5	3	9	9	9	3	3	3	3	3
1																													
2																													
3																													
4																													
5																													
6																													
RIC =																													

PROCESSING QUEUE

No	ACT	RES	TF	SQS
1	G	9	12	3
2	B	7	8	2
3	I	6	1	4
4	J	5	6	4
5	E	4	8	3
6	D	4	13	2

	G Assign.	$\sum RS(G)$	B Assign.	$\min \sum RS(B)$	I Assign.	$\min \sum RS(I)$	J Assign.	$\min \sum RS(J)$	Σ
ACT G	9-10	4	7-8	4	22-25	3	20-22	13	24
AC = 11	10-11	4	7-8	4	22-25	3	20-22	13	24
DUR = 2	11-12	4	7-8	4	22-25	3	20-22	13	24
	12-13	4	7-8	4	22-25	3	20-22	13	24
Assign to day 9-10	13-14	4	7-8	4	22-25	3	20-22	13	24
	14-15	4	7-8	4	22-25	3	20-22	13	24
	15-16	4	7-8	4	22-25	3	20-22	13	24
	16-17	4	7-8	4	22-25	3	20-22	13	24
	17-18	7	7-8	4	22-25	3	20-22	13	27
	18-19	10	7-8	4	22-25	3	20-22	13	30
	19-20	10	7-8	4	22-25	3	21-23	17	34
	20-21	10	7-8	4	22-25	3	24-26	21	38
	21-22	10	7-8	4	23-26	3	24-26	21	38

SQS	RES	AC	JC	DUR	ACT	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	5	0	0	6	A																								
2	2	0	0	11	C	1	1	1	1	1	1	1	1	1	1	1													
3	5	0	0	4	F												1	1	1	1									
4	3	0	0	5	H																1	1	1	1	1				
5	0	0	0	4	K																					1	1	1	1
5	3	0	0	4	L																					1	1	1	1
6	7	0	0	7	M																								
2	7	0 8		2	B			0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2														
2	2			3	D																								
		13																											
3	4			5	E																								
		8																											
3	9		1	2	G			1 2	1 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2								
4	6			4	I																								
		1																											
4	5			3	J																								
		6																											
No			assign		RS	2	2	2	2	2	2	2	2	2	2	2	5	5	5	5	3	9	9	9	3	3	3	3	3
1			9-10		G			11	11																				
2					B																								
3					I																								
4					J																								
5					E																								
6					D																								
RIC =																													

ACT B
 AC = 0
 DUR = 2
 Assign to day 7-8

B						Σ
Assign.	$\Sigma RS(B)$					
7-8	4					4

ACT I
 AC = 1
 DUR = 4
 Assign to day 22-25

I		D				Σ
Assign.	$\Sigma RS(I)$	Assign.	$\min \Sigma RS(D)$			
22-25	3	11-13	6			9
23-26	3	11-13	6			9

ACT J
 AC = 6
 DUR = 3
 Assign to day 18-20

J						Σ
Assign.	$\Sigma RS(J)$					
18-20	15					15
19-21	15					15
20-22	19					19
21-23	23					23
22-24	27					27
23-25	27					27
24-26	21					21

ACT E
 AC = 6
 DUR = 3
 Assign to day 11-15

E Assign.	$\sum RS(E)$					Σ
9-13	28					28
10-14	19					19
11-15	10					10
12-16	10					10
13-17	10					10
14-18	18					18
15-19	26					26
16-20	34					34
17-21	37					37
D Assign.	$\sum RS(D)$					Σ
7-9	29					29
8-10	31					31
9-11	28					28
10-12	23					23
11-13	18					18
12-14	18					18
13-15	18					18
14-16	14					14
15-17	10					10
16-18	14					14
17-19	22					22
18-20	30					30
19-21	25					25

ACT D
 AC = 12
 DUR = 3
 Assign to day 15-17

SQS	RES	AC	JC	DUR	ACT	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	5	0	0	6	A																								
2	2	0	0	11	C	1	1	1	1	1	1	1	1	1	1	1													
3	5	0	0	4	F												1	1	1	1									
4	3	0	0	5	H																1	1	1	1	1				
5	0	0	0	4	K																					1	1	1	1
5	3	0	0	4	L																					1	1	1	1
6	7	0	0	7	M																								
2	7	0 8		2	B	1 2	1 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2														
2	2	12 13		3	D	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	1 2	1 2	1 2	0 2	0 2	0 2	0 2	0 2								
3	4	8		5	E			0 2	0 2	1 2	1 2	1 2	1 2	1 2	0 2	0 2	0 2	0 2	0 2	0 2									
3	9	11		2	G			1 2	1 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2									
4	6	1		4	I																1 2	1 2	1 2	1 2	0 2				
4	5	6		3	J												1 2	1 2	1 2	0 2	0 2	0 2	0 2	0 2	0 2				
No			assign		RS	2	2	2	2	2	2	2	2	2	2	2	5	5	5	5	3	9	9	9	3	3	3	3	3
1			9-10		G			11	11																				
2			7-8		B	9	9																						
3			22-25		I																9								
4			18-20		J												10	10	10										
5			11-15		E					6	6	6	6	6															
6			15-17		D									10	6	6													
RIC = 1.120						9	9	11	11	6	6	6	6	10	6	6	10	10	10	5	9	9	9	9	3	3	3	3	3

Solution 10.2 Problem 10.2 Leveling the following network using PACK

Activity	Predecessors	Duration	Resource Rate
A		2	7
B	A	3	2
C	A	5	2
D	B	2	3
E	B	4	1
F	C	3	2
G	C	10	2
H	E	4	5
I	E,F	2	3
J	D,H,I,G	3	6

SQS	RES	AC	JC	DUR	ACT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	7	0	0	6	A	1	1																		
2	2	0	0	11	C			1	1	1	1	1													
3	2	0	0	4	G								1	1	1	1	1	1	1	1	1	1			
4	6	0	0	5	J																		1	1	1
2	2	1	5	3	B			1	1	1	0	0	0	0											
								2	2	2	2	2	2	2											
3	3	1	10	2	D						0	0	0	0	0	0	0	0	1	1	0	0			
											2	2	2	2	2	2	2	2	2	2	2	2	2		
3	1	1	4	4	E						1	1	1	1	0	0	0	0							
											2	2	2	2	2	2	2	2							
3	2	1	5	3	F								1	1	1	0	0	0	0	0					
													2	2	2	2	2	2	2	2					
4	5	1	4	4	H										1	1	1	1	0	0	0	0			
															2	2	2	2	2	2	2	2			
4	3	1	5	2	I											0	0	0	0	0	0	1	1		
																2	2	2	2	2	2	2	2		
No					RS	5	5	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	6	6	6
1					H										7	7	7	7							
					E						3	3	3	3											
2					I																5	5			
3					D														5	5					
4					B			4	4	4															
5					F								5	5	9										
						7	7	4	4	4	3	3	5	5	9	7	7	7	5	5	5	5	6	6	6

RIC = 1.0744

PROCESSING QUEUE

No	ACT	RES	TF	SQS
1	H	5	4	4
2	I	3	5	4
3	D	3	10	3
4	B	2	4	3
5	F	2	5	3
6	E	1	4	3

ACT	H	H	E		
AC =	4	Assign.	Assign.	$\min \sum RS(J)$	Σ
DUR =	4	10-13	6-9	8	16
Assign to day	10-13	11-14	6-9	8	16
		12-15	6-9	8	16
		13-16	6-9	8	16
		14-17	6-9	8	16

Since E is a predecessor of H and it must occupied day 6-9 no matter what, you can assign activity E into the resource profile right the way.

ACT	E	E	
AC =	0	Assign.	$\sum RS(J)$
DUR =	4	6-9	8
Assign to day	6-9		8

ACT	I	I		F	
AC =	5	Assign.	$\sum RS(J)$	Assign.	$\min \sum RS(J)$
DUR =	2	11-12	14	8-10	13
Assign to day	16-17	12-13	14	8-10	13
		13-14	9	8-10	13
		14-15	4	8-10	13
		15-16	4	8-10	13
		16-17	4	13-15	11
					Σ
					27
					27
					22
					17
					17
					15

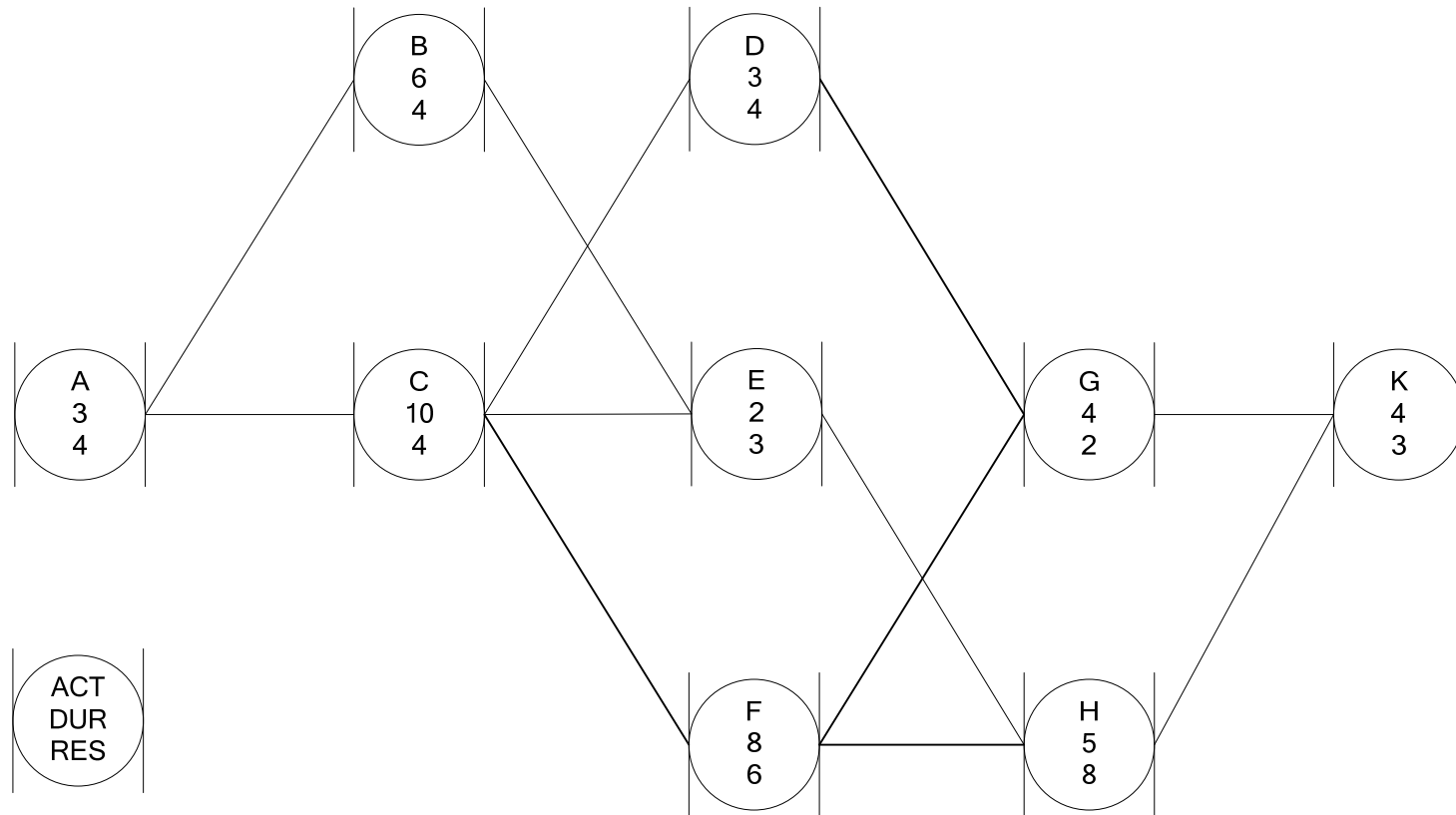
ACT	D	D		Σ
AC =	10	Assign.	$\sum RS(J)$	
DUR =	2	6-7	6	6
Assign to days	14-15	7-8	6	6
		8-9	6	6
		9-10	10	10
		10-11	14	14
		11-12	14	14
		12-13	14	14
		13-14	9	9
		14-15	4	4
		15-16	7	7
		16-17	10	10

ACT	B	B		Σ
AC =	0	Assign.	$\sum RS(J)$	
DUR =	3	3-5	6	6
Assign to days	3-5			

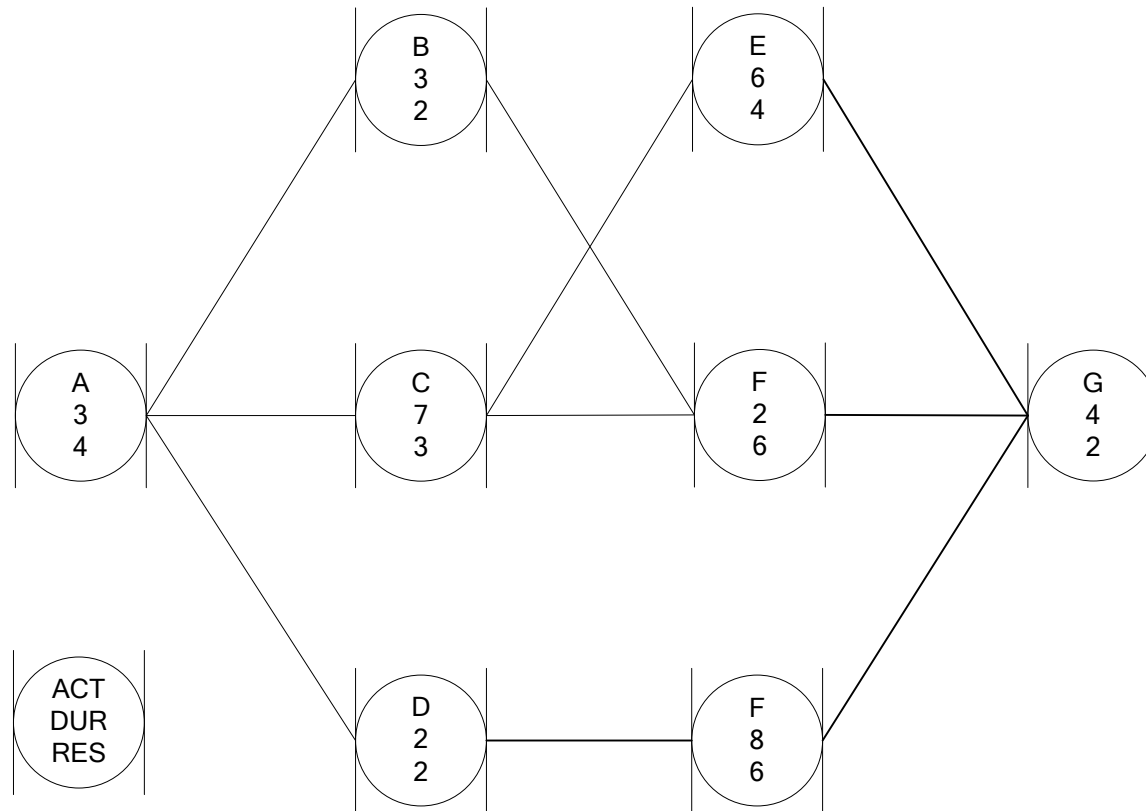
ACT F
 AC = 5
 DUR = 3
 Assign to days 8-10

F Assign.	$\sum RS(J)$	Σ
8-10	13	13
9-11	17	17
10-12	21	21
11-13	21	21
12-14	19	19
13-15	17	17

Problem 10.3



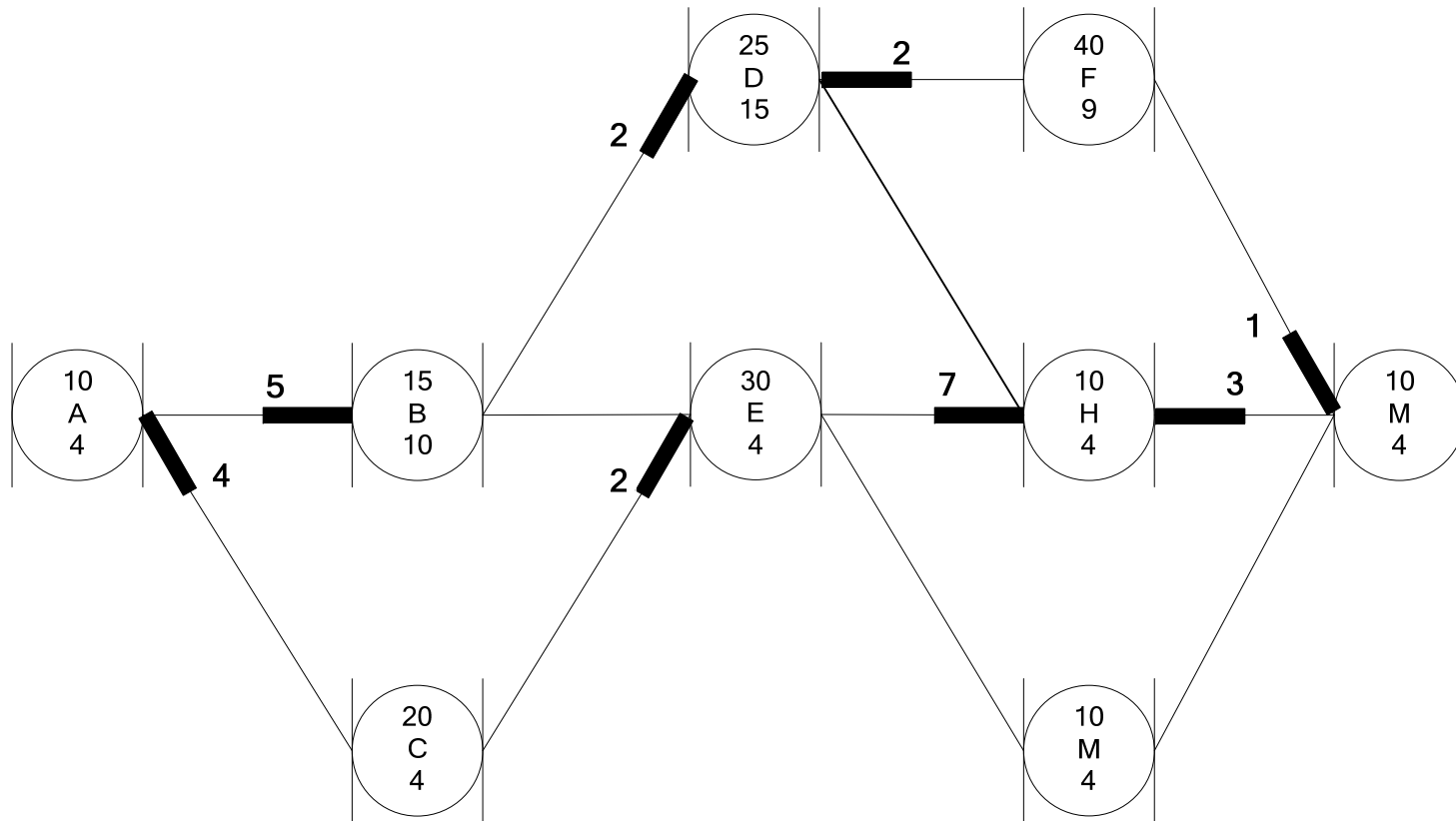
Problem 10.4



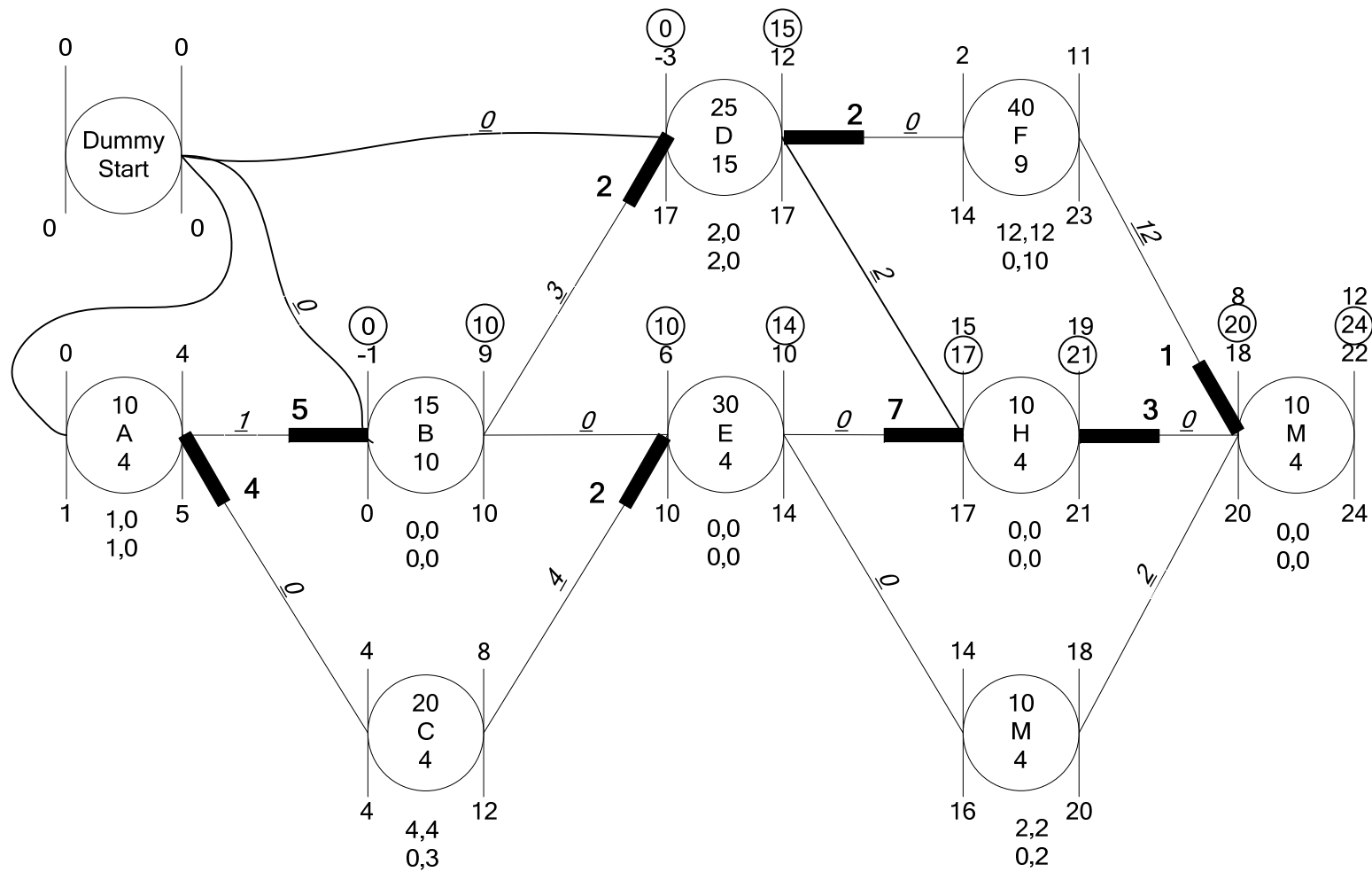
11.OVERLAPPING NETWORKS

11. OVERLAPPING NETWORK

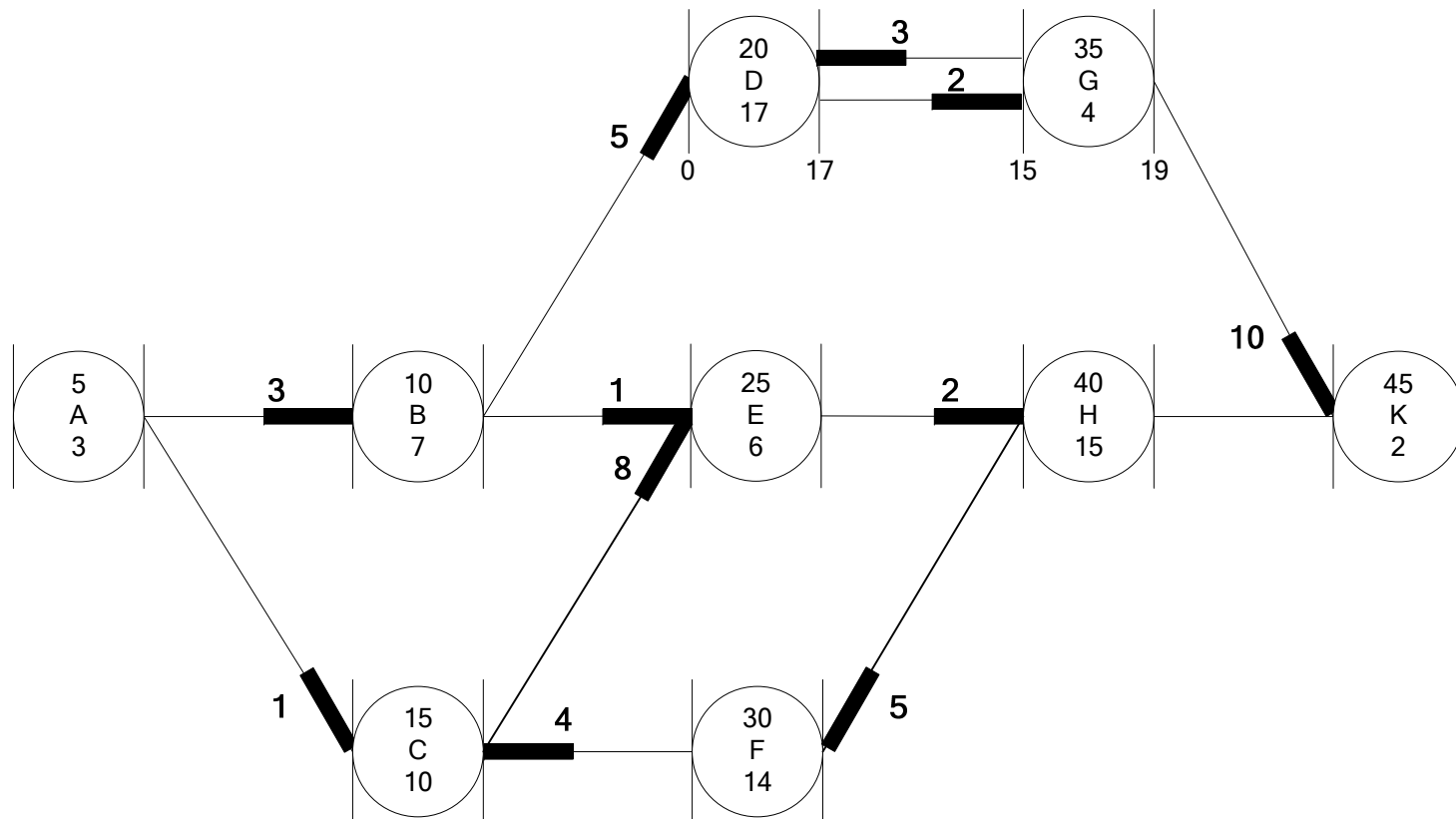
Problem 11.1 Calculate ESD, EFD, TF, and FF



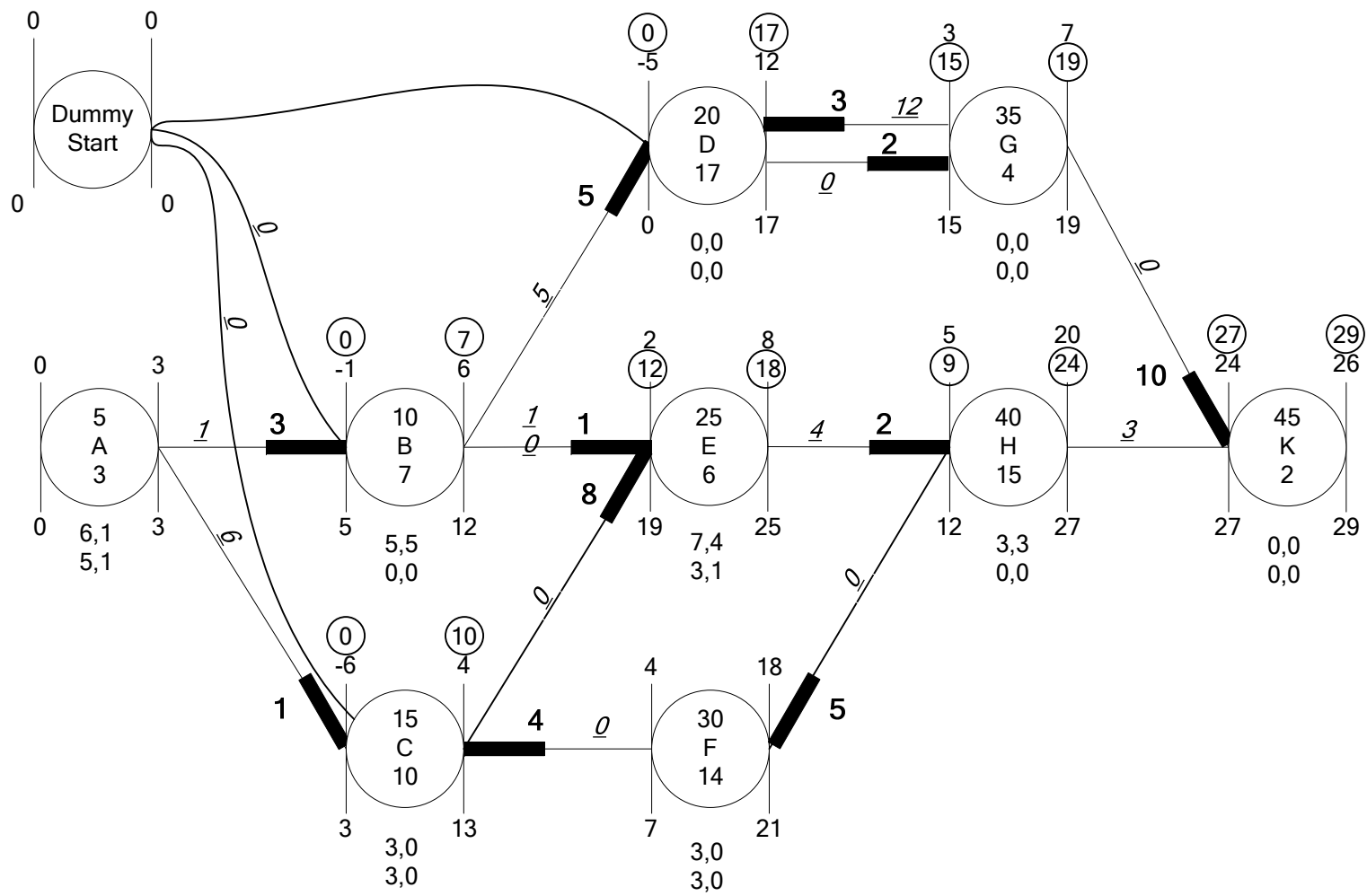
Solution 11.1



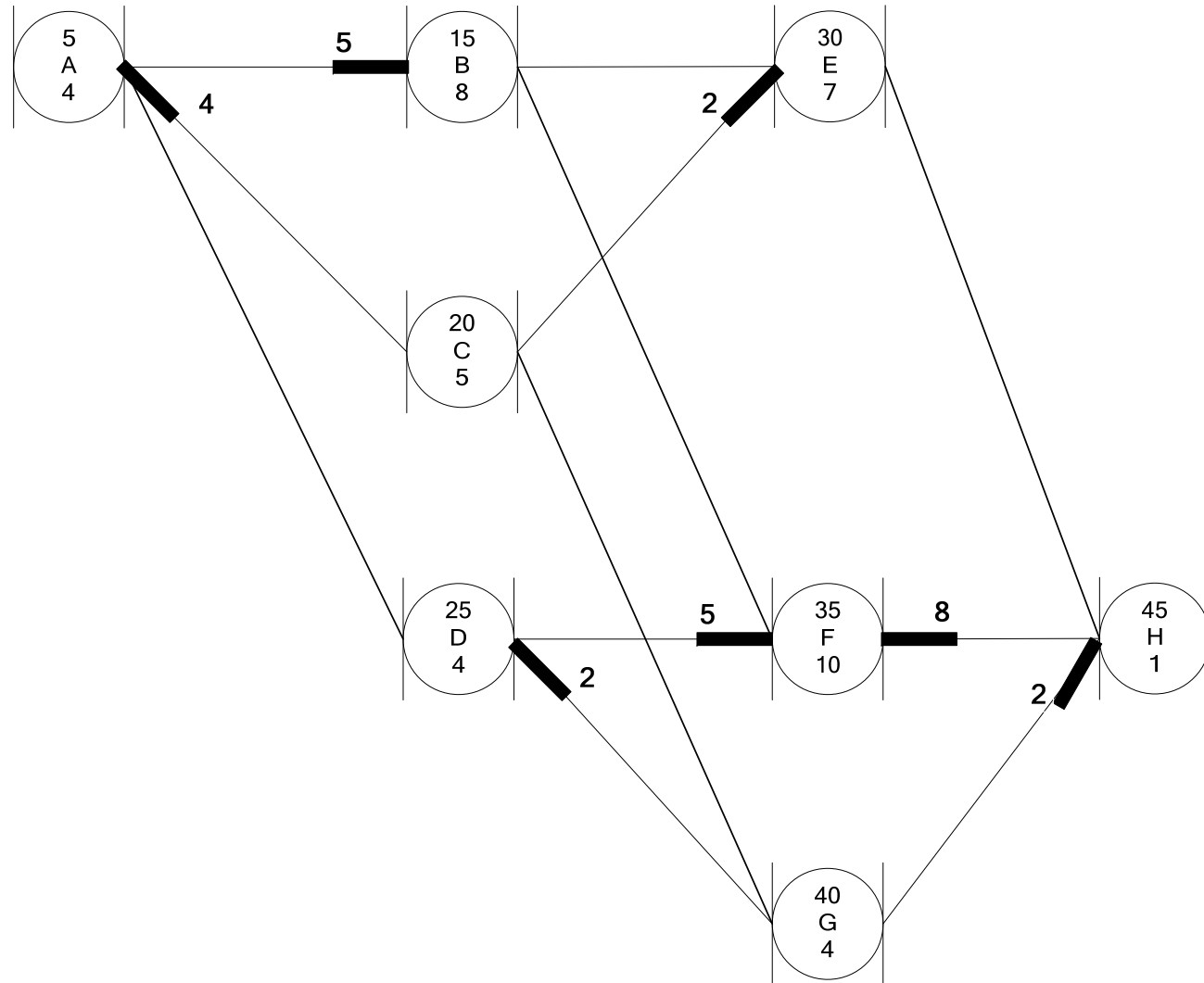
Problem 11.2



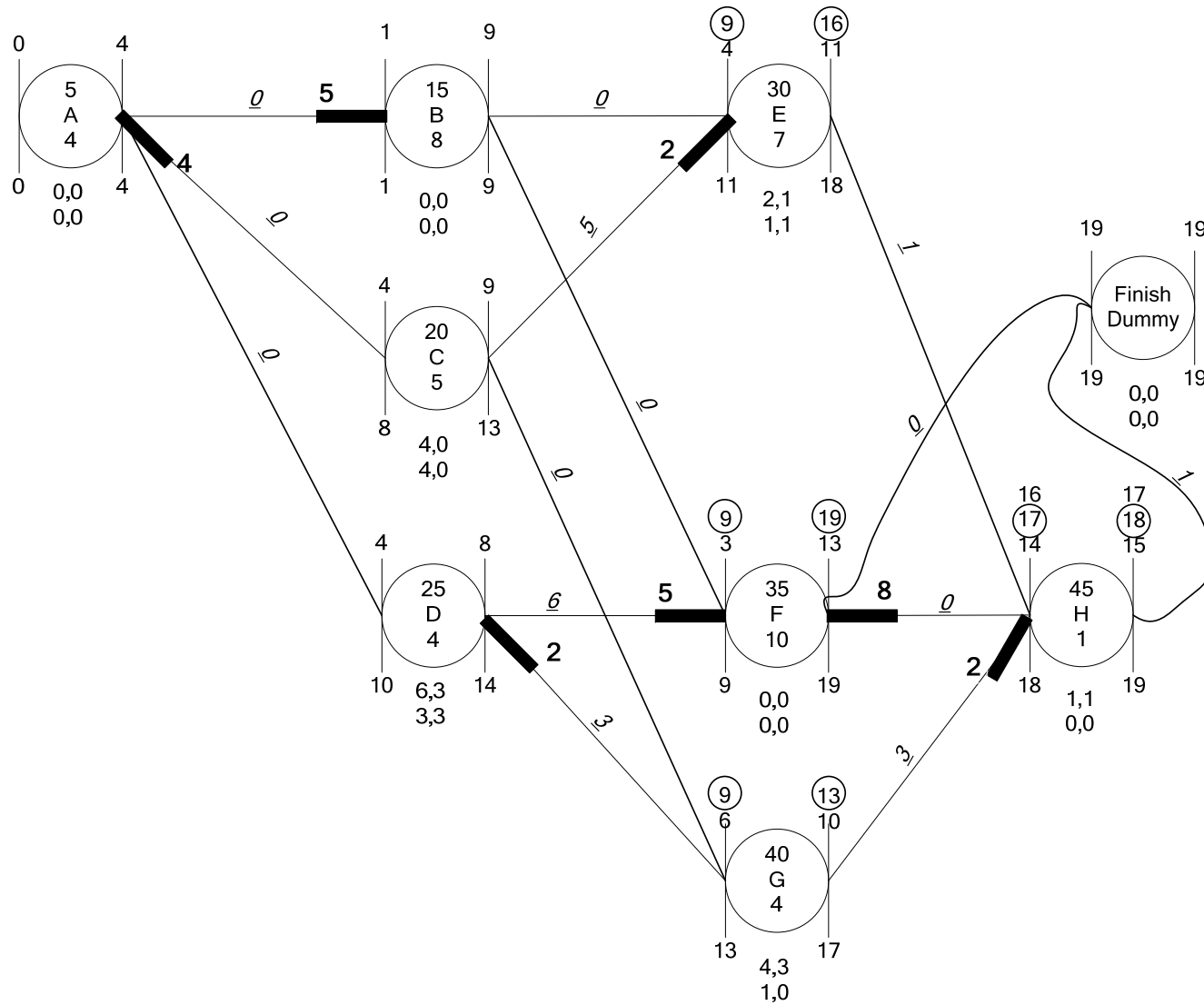
Solution 11.2



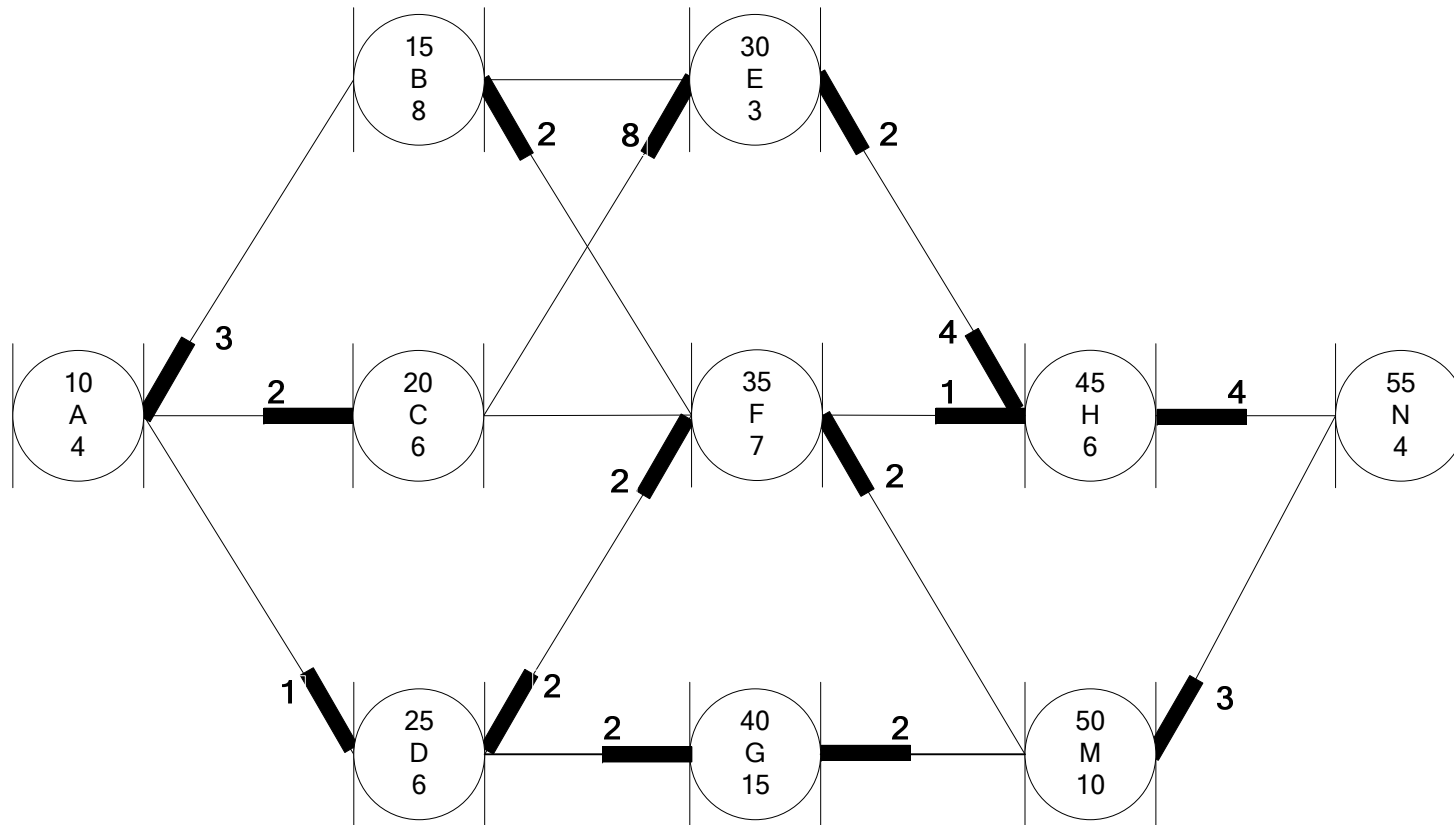
Problem 11.3



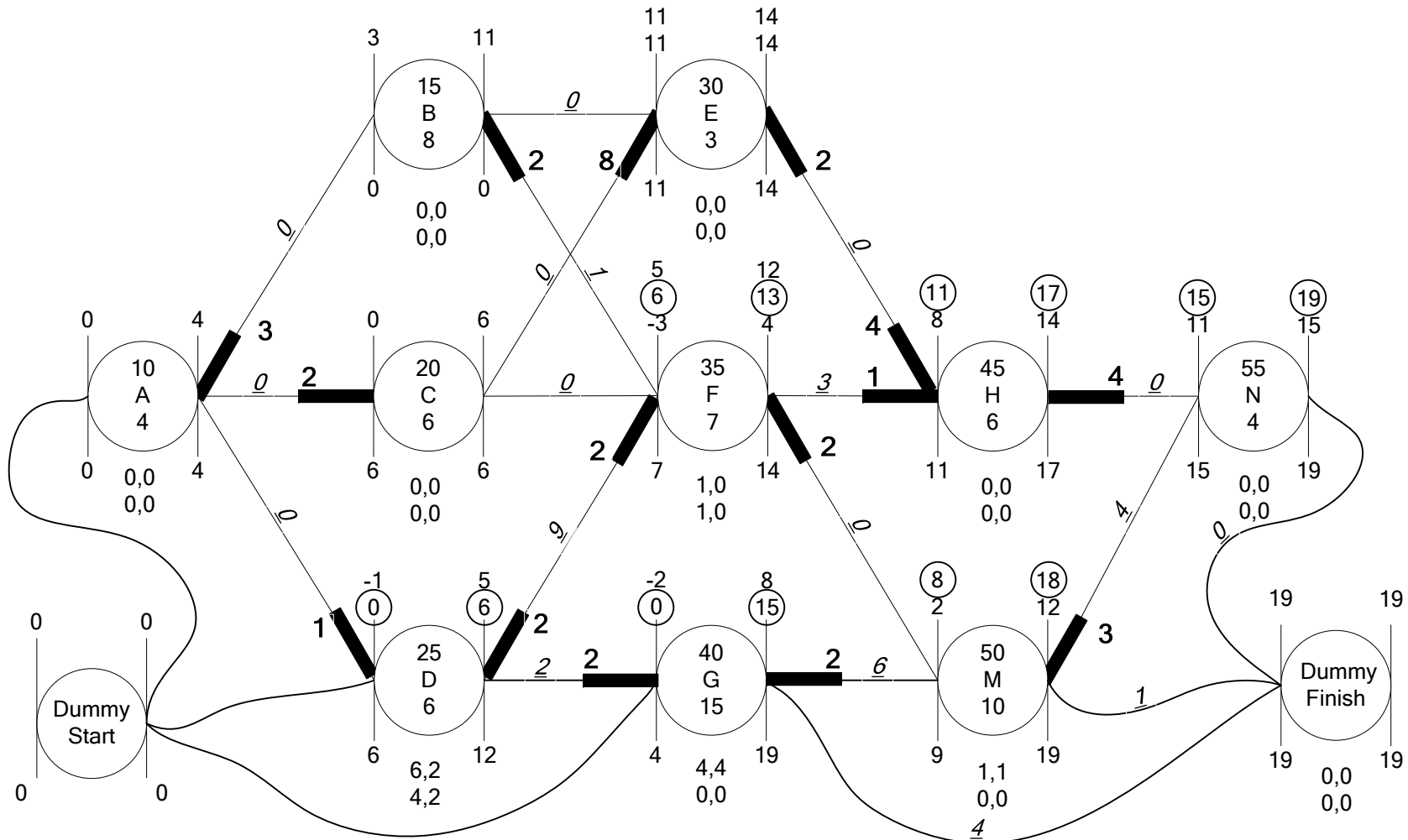
Solution 11.3



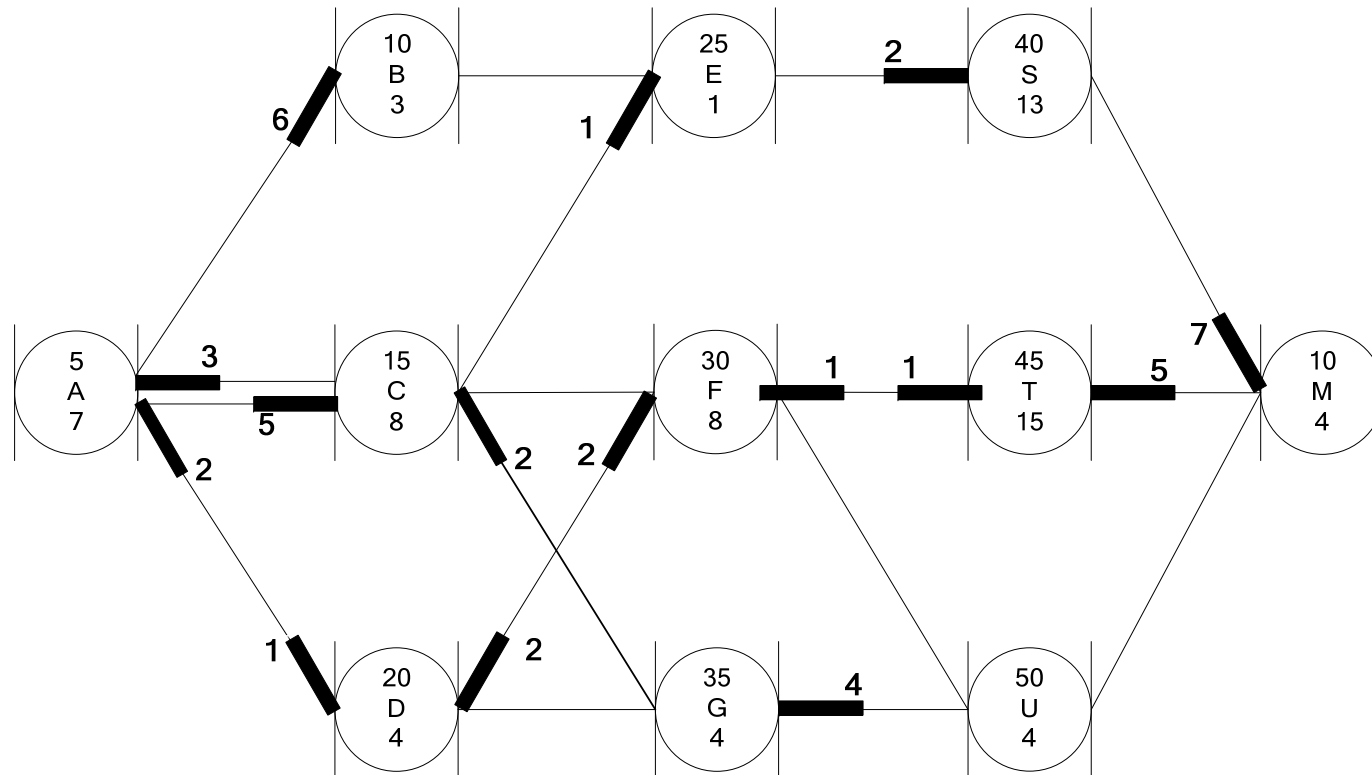
Problem 11.4



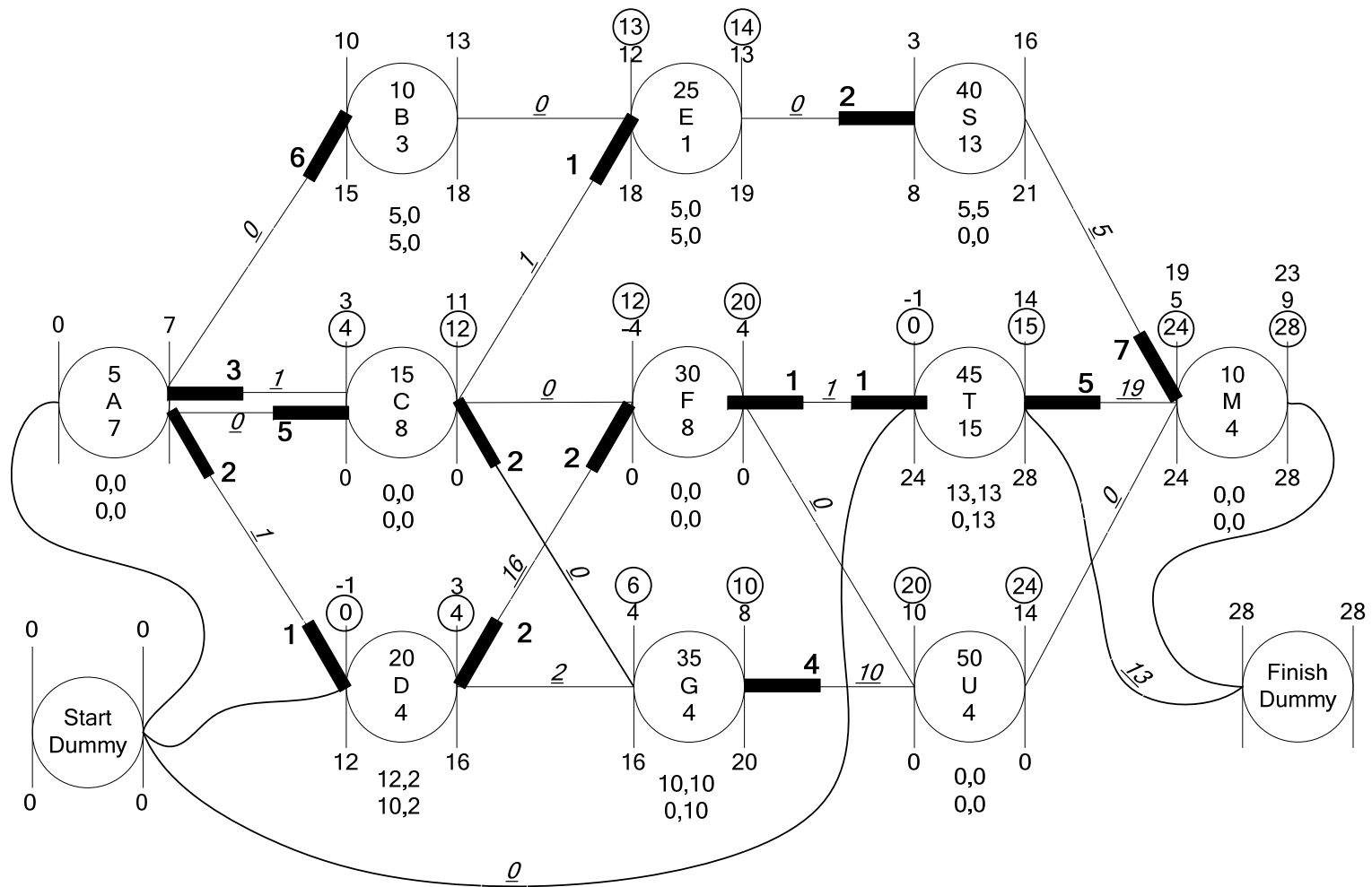
Solution 11.4



Problem 11.5



Solution 11.5



12.REPETITIVE SCHEDULING METHOD

REPETITIVE SCHEDULING METHOD (RSM)

Problem 12.1

This project consists of 5 units requiring 7 activities.

Construct RSM diagram of the following project

Determine critical activities

Identify controlling sequence

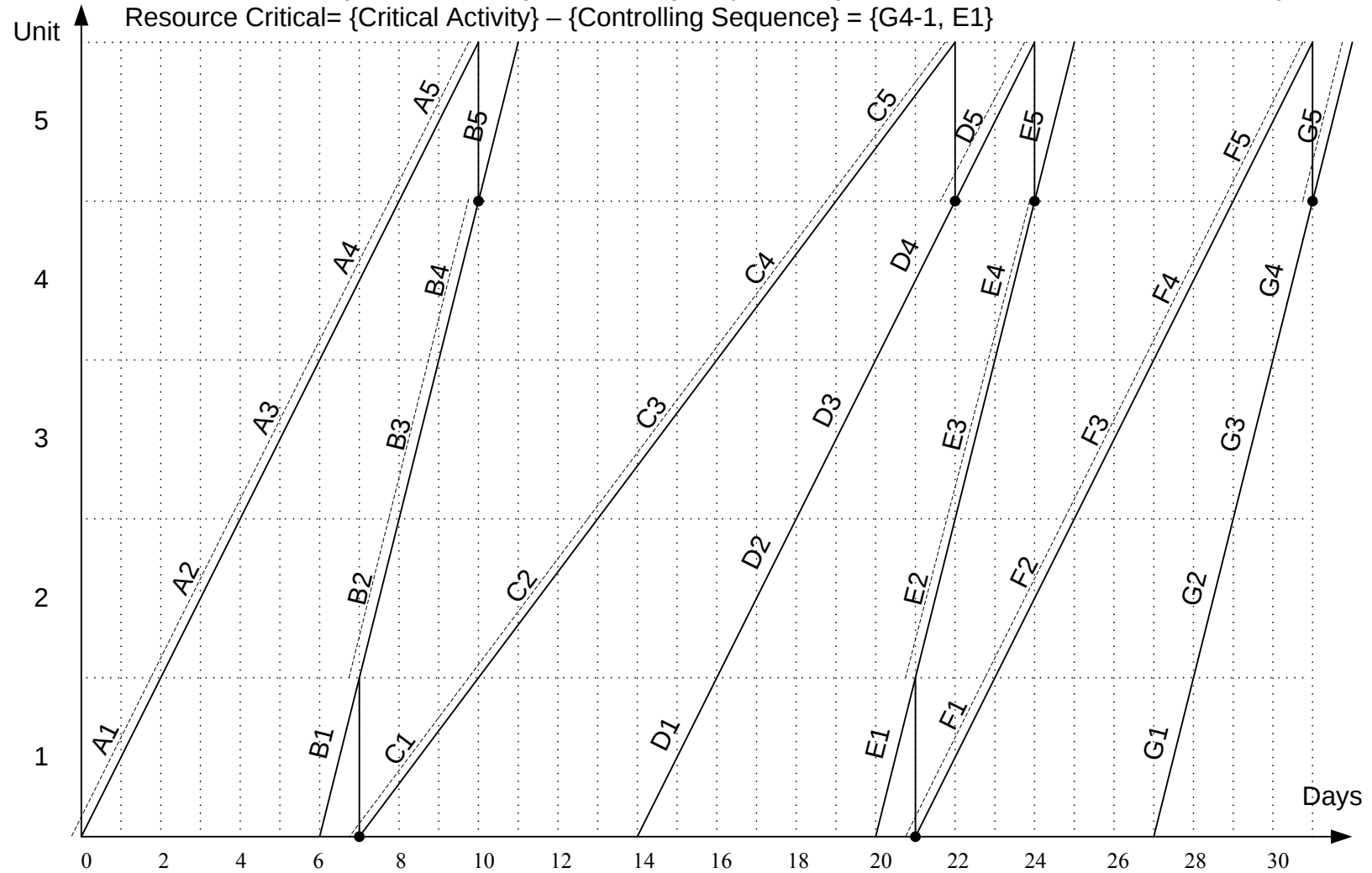
Identify resource critical activities

	Activity	Predecessors	Duration/Unit
1	A		2
2	B	A	1
3	C	B	3
4	D	C	2
5	E	D	1
6	F	E	2
7	G	F	1

Solution 12.1

Critical Activities= {G5-1, F5-1, E1} , Controlling Sequence= {G5, F5-1, E2-4, D5, C5-1, B2-4, A5-1}

Resource Critical= {Critical Activity} – {Controlling Sequence} = {G4-1, E1}



Problem 12.2

This project consists of 3 units requiring 7 activities.

Construct RSM diagram of the following project

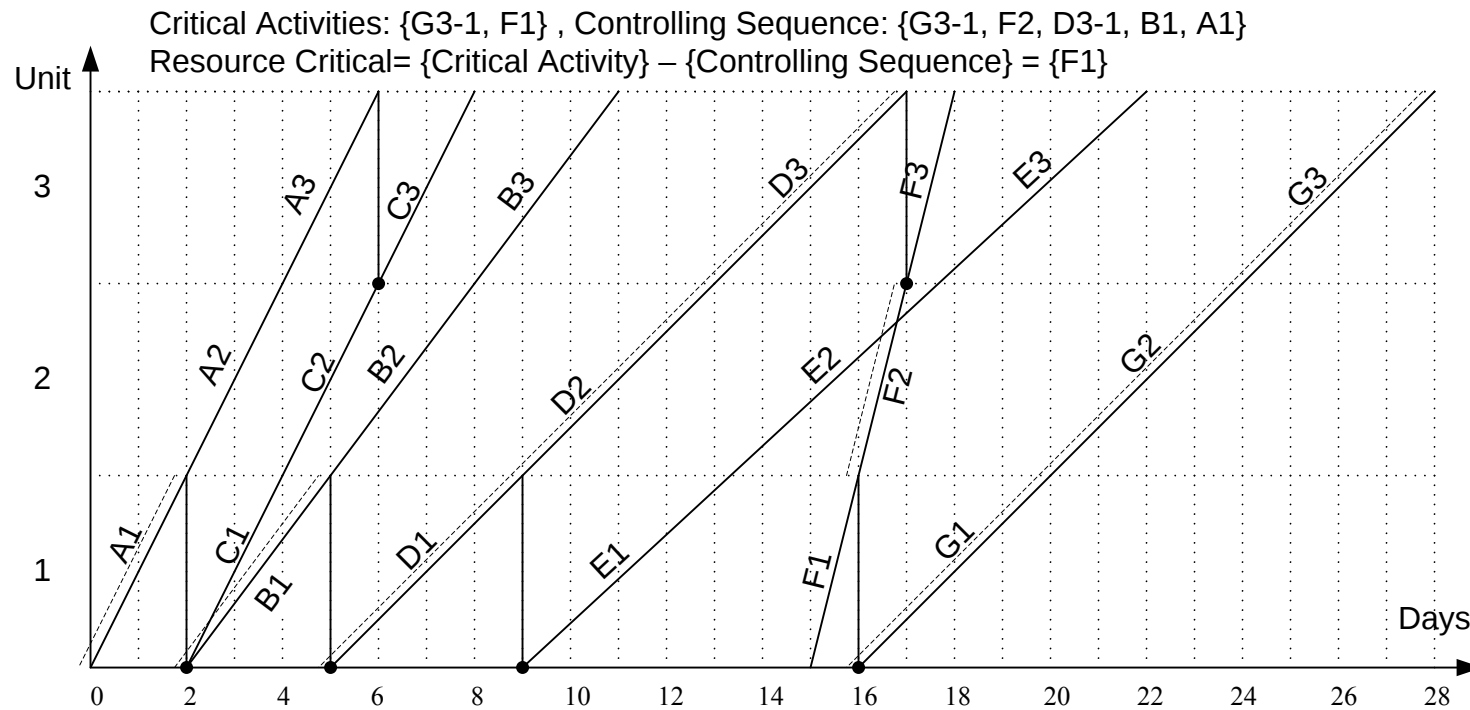
Determine critical activities

Identify controlling sequence

Identify resource critical activities

	Activity	Predecessors	Duration/Unit
1	A		2
2	B	A	3
3	C	A	2
4	D	B, C	4
5	E	D	4.333
6	F	D	1
7	G	E, F	4

Solution 12.2



Problem 12.3

This project consists of 5 units requiring 7 activities.

Construct RSM diagram of the following project

Determine critical activities

Identify controlling sequence

Identify resource critical activities

	Activity	Predecessors	Duration/Unit
1	A		3
2	B	A	5
3	C	A	1
4	M	C	2
5	D	B	2
6	E	M, D	2
7	F	E	3

Solution 12.3

