

Trigonometric Identities & Formulas

Tutorial Services – Mission del Paso Campus

Reciprocal Identities

$$\sin x = \frac{1}{\csc x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\cot x = \frac{1}{\tan x}$$

Ratio or Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \cos x \tan x$$

$$\cos x = \sin x \cot x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Note: there are only three, basic Pythagorean identities, the other forms are the same three identities, just arranged in a different order.

Pythagorean Identities in Radical Form

$$\sin x = \pm \sqrt{1 - \cos^2 x}$$

$$\tan x = \pm \sqrt{\sec^2 x - 1}$$

$$\cos x = \pm \sqrt{1 - \sin^2 x}$$

Confunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Odd-Even Identities

Also called negative angle identities

$$\sin(-x) = -\sin x \quad \csc(-x) = -\csc x$$

$$\cos(-x) = \cos x \quad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \quad \cot(-x) = -\cot x$$

$$\text{Phase Shift} = \frac{-c}{b}$$

$$\text{Period} = \frac{2\pi}{b}$$

Sum and Difference Formulas/Identities

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

How to Find Reference Angles

Step 1: Determine which quadrant the angle is in

Step 2: Use the appropriate formula

Quad I = is the angle itself

Quad II = $180 - \theta$ or $\pi - \theta$

Quad III = $\theta - 180$ or $\theta - \pi$

Quad IV = $360 - \theta$ or $2\pi - \theta$

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x} \quad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x} \quad \cot x = \frac{1}{\tan x}$$

Ratio or Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \cos x \tan x \quad \cos x = \sin x \cot x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

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Confunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Odd-Even Identities

Also called negative angle identities

$$\sin(-x) = -\sin x \quad \csc(-x) = -\csc x$$

$$\cos(-x) = \cos x \quad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \quad \cot(-x) = -\cot x$$

Sum and Difference Formulas - Identities

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

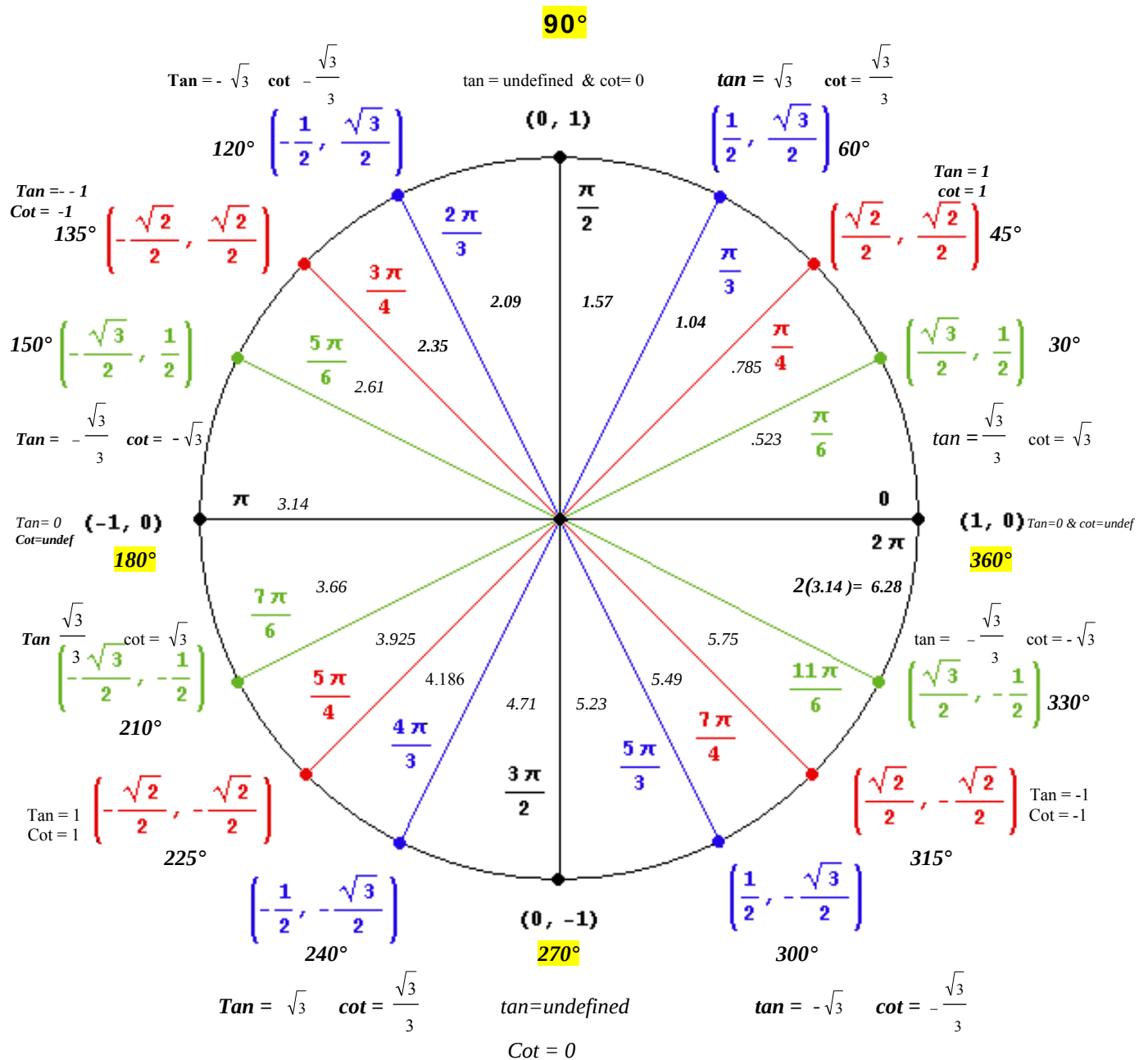
$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

The Unit Circle



Definition of Trigonometric Functions concerning the Unit Circle

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$

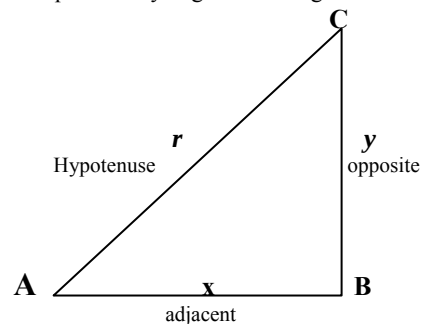
Right Triangle Definitions of Trigonometric Functions

Note: sin & cos are complementary angles, so are tan & cot and sec & csc, and the sum of complementary angles is 90 degrees.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \quad \csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \quad \sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} \quad \cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$



Adjacent = is the side adjacent to the angle in consideration. So if we are considering Angle A, then the adjacent side is CB

Trigonometric Values of Special Angles

Degrees	0°	30°	45°	60°	90°	180°	270°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
sinθ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tanθ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	0	undefined

To Convert Degrees to Radians, Multiply by $\frac{\pi \text{ rad}}{180 \text{ deg}}$

To Convert Radians to Degrees, Multiply by $\frac{180 \text{ deg}}{\pi \text{ rad}}$

Vocabulary

- Cotangent Angles - are two angles with the same terminal side
- Reference Angle - is an acute angle formed by terminal side of angle(α) with x-axis

Double Angle Identities

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Half Angle Identities

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

Power Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2} [\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u + v) - \sin(u - v)]$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right)$$

$$\sin x - \sin y = 2 \cos \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$$

$$\cos x + \cos y = 2 \cos \left(\frac{x + y}{2} \right) \cos \left(\frac{x - y}{2} \right)$$

$$\cos x - \cos y = -2 \sin \left(\frac{x + y}{2} \right) \sin \left(\frac{x - y}{2} \right)$$

Law of Sines

Solving Oblique Triangles using sine: AAS, ASA, SSA, SSS, SAS

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

Cosine: SAS, SSS

Standard Form

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Alternative Form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Finding the Area of non-90degree Triangles

Area of an Oblique Triangle

$$\text{area} = \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B$$

Heron's Formula

Step 1: Find "s" $s = \frac{(a + b + c)}{2}$

Step 2: Use the formula $\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$